Online Appendix

Equilibrium effects of tax exemptions for low pay

Luke Haywood & Michael Neumann

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Part I

Appendices

A Stylized model

The objective of this section is to show how a notch in the budget set of one group of workers affects the earnings distribution of all workers. Two simplifying assumptions enable us to solve the model analytically (both are relaxed in the main text). First, job offers are assumed to have the same hours requirements. This implies that utility increases monotonously in net earnings, which thus completely describe a job offer. Second, we focus only on the most important source of heterogeneity across workers, splitting them into two groups: The first group, full-time workers seeking a second job (population n^s), has no discontinuous incentive to locate at or below the threshold, since no tax deductions apply. For the second group seeking a small job as a first job (population n^f) we assume that the implicit marginal tax rate is so high that they only accept job offers with earnings up to the threshold. Importantly, the prominence of type-f workers generates a strong incentive for firms to over-proportionally offer such jobs. With these constraints in mind we can deduct how many workers different job offers will attract. This allows us to determine the equilibrium firm size distribution $l(z)$, critical in determining firms' optimal job offers in equilibrium.

Worker mobility

In equilibrium, the flows of workers of each type j moving in and out of small jobs must balance:

$$\delta^j(n^j - u^j) = \begin{cases} \lambda^j u^j & \text{for } j = s \\ \lambda^j u^j F(z^*) & \text{for } j = f \end{cases} \tag{1}$$

where u^j denotes the number of small-job-seeking type-j workers and $\kappa^j \equiv \frac{\lambda^j}{\delta^j}$. The flows differ across types since in this stylized model, type-f workers do not accept jobs with earnings $z > z^*$. The measures of small-job-seekers are thus:

$$u^j = \begin{cases} \frac{n^j}{1+\kappa^j} & \text{for } j = s \\ \frac{n^j}{1+\kappa^j \ F(z^*)} & \text{for } j = f \end{cases} \tag{2}$$

Similarly, in the steady-state the flow of small-job-seekers into small jobs with earnings no greater than z must equal the measure of small job separations. The latter comprises employees losing their small job with earnings no greater than z as well as workers moving to more attractive small jobs (the left-hand side of equation (3) with $G^j(.)$ denoting the distribution of realized earnings for type-j workers).

$$\lambda^j F(z) u^j = \begin{cases} [\delta^j + \lambda^j (1 - F(z))] G^j(z)(n^j - u^j) & \text{for } j = s \\ [\delta^j + \lambda^j (F(z^*) - F(z))] G^j(z)(n^j - u^j) & \text{for } j = f \ \ \& \ \ z \leq z^* \end{cases} \tag{3}$$

Firm size

In the steady-state the number of workers of type j employed at a firm offering jobs with earnings z can be expressed by equation (4),

$$l^j(z) = \lim_{\epsilon \to 0} \frac{(G^j(z) - G^j(z-\epsilon))(n^j - u^j)}{F(z) - F(z-\epsilon)} \quad \text{for } j \in (s, f) \tag{4}$$

We now derive the number of workers a firm can attract by offering earnings z, based on equations (1), (3) and (4). We start with workers who also hold a full-time job:

$$l^s(z) = \frac{\frac{F(z)}{1+\kappa^s(1-F(z))} - \frac{F(z-\epsilon)}{1+\kappa^s(1-F(z-\epsilon))}}{F(z) - F(z-\epsilon)} (n^s - u^s)$$

which simplifies to

$$= \frac{n^s \kappa}{(1 + \kappa^s(1 - F(z)))(1 + \kappa^s(1 - F(z-\epsilon)))}.$$

In this stylized model, workers without full-time jobs only accept jobs if $z \leq z^*$. We thus have

$$l^f(z) = \frac{\frac{F(z)}{(1+\kappa^f(F(z^*)-F(z)))F(z^*)} - \frac{F(z-\epsilon)}{(1+\kappa^f(F(z^*)-F(z-\epsilon)))F(z^*)}}{F(z) - F(z-\epsilon)} (n^f - u^f)$$

which simplifies to

$$= \frac{n^f \kappa^f}{(1 + \kappa^f(F(z^*) - F(z)))(1 + \kappa^f(F(z^*) - F(z-\epsilon)))}$$

The total firm size is then the sum of the number of workers of each type, i.e.

$$l(z) = l^s(z) + l^f(z)$$

$$= \begin{cases} \frac{n^s \kappa^s}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))} + \frac{n^f \kappa^f}{(1+\kappa^f(F(z^*)-F(z)))(1+\kappa^f(F(z^*)-F(z-\epsilon)))} & \forall z \leq z^* \\ \frac{n^s \kappa^s}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))} & \forall z > z^* \end{cases} \tag{5}$$

Equilibrium job offer distribution

Proposition (A.I) *If we observe offers above z^*, there must be a mass point of job offers at z^*. The earnings offer distribution above z^* is continuous up to the highest earnings offer, $\bar{z}$.*

Intuitively, the logic of proposition (A.I) is that equal profits at and marginally above the threshold can only hold when the loss of type-f workers is balanced by a discontinuously large number of type-s workers that can be attracted by exceeding the threshold. This requires a mass point at z^*. Any earnings offer above the threshold will be attractive to all type-s workers currently earning z^* (i.e. located at the mass point). As we observe positive earnings mass above the threshold in the data (see section 4.3 in the main test), proposition (A.I) implies that there must be a mass point at $z = z^*$ (i.e. that $f(z^*) > 0$).

Formally, assume there exists no mass point (i.e. $f(z^*) = 0$), then the offer distribution for $z < z^*$ is continuous and profits at the threshold are

$$\pi(z^*) = (p - z^*)\left[\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)))^2} + n^0\kappa^0\right]. \tag{6}$$

Profits associated with offering earnings slightly above the threshold (for $\epsilon \to 0$) are:

$$\begin{aligned}\pi(z^*+\epsilon) &= (p-(z^*+\epsilon))\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*+\epsilon)))(1+\kappa^s(1-F(z^*-\epsilon+\epsilon)))} \\ &= (p-z^*)\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)))^2}\end{aligned} \tag{7}$$

Assuming $f(z^*) = 0$, we have $\pi(z^*+\epsilon) < \pi(z^*)$ which is not consistent with our assumption of equal profits. Thus if we assume that there is no mass point in the earnings distribution, then there must be a gap in the earnings distribution above the threshold. We can now ask: If there is a gap in the earnings distribution, will there be any earnings level $z' > z^* + \epsilon$ where the equal profit condition holds again? Given that we are assuming no mass point and a gap in the interval $z \in (z^*, z')$, we have $F(z') = F(z^*)$, which can be used to derive equation (8):

$$\begin{aligned}\pi(z') &= (p-z')\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z')))(1+\kappa^s(1-F(z'-\epsilon)))} \\ &= (p-(z'))\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)))^2}\end{aligned} \tag{8}$$

As $(p-z') < (p-z^*)$, it follows that $\pi(z') < \pi(z^*)$ implying that no job with earnings $z > z^*$ will be offered if there is no mass point at z^*. If we observe any earnings above the threshold, this is only consistent if there is a mass point at z^*. This completes the proof

of the first part of proposition (A.I).

Allowing for a mass point at z^*, $\frac{\partial \pi(z^*)}{\partial f(z^*)} < 0$ and $\frac{\partial \pi(z^*+\epsilon)}{\partial f(z^*)} = 0$ imply that there might be a value for $f(z^*)$ for which the equal profit condition between z^* and $z^* + \epsilon$ holds ($\pi(z^* + \epsilon) = \pi(z^*)$). For earnings $z' \in [z^* + \epsilon, \overline{z}]$, the usual trade-off between profit per workers and firm size ensures that the equilibrium offer distribution is continuous in that interval and determined by $\pi(z') = \pi(z^* + \epsilon)$.

Proposition (A.II) *If there is a mass point at z^*, there will be a gap in the offer distribution just below the threshold. The gap may be such that there are no offers below z^* in equilibrium. If there are earnings below z^*, the earnings offer distribution will then be continuous between the left limit of the gap, z'', and reservation earnings, z^r.*

To understand this proposition, note that a mass point in our setting implies that any job offer with earnings just below the mass point ($z^* - \varepsilon$) would generate less profits, since margins per worker are only slightly higher, but firm size will be discontinuously lower since there is a mass of firms (offering z^*) that can poach any worker employed at earnings $z^* - \varepsilon$. This raises the second question, namely whether any offers with earnings below z^* are generated in equilibrium. This may or may not be the case depending on whether an increase in profit per worker is able to balance the lower capacity to attract workers (and thus smaller firm size) compared to locating at the mass point.

We first show that a mass point in the wage offer distribution is only consistent with equal profits if there is a gap in the wage offer distribution. We compare profits $\pi(z^*)$ with profits $\pi(z^* - \epsilon)$. Profits of a job offer with earnings at the threshold are given by

$$\begin{aligned}
\pi(z^*) &= (p - z^*) l(z^*) \\
&= (p - z^*) \left(\frac{n^s \kappa^s}{(1 + \kappa^s (1 - F(z^*)))(1 + \kappa^s (1 - F(z^* - \epsilon)))} + \frac{n^0 \kappa^0}{(1 + \kappa^0 (F(z^*) - F(z^*)))(1 + \kappa^0 (F(z^*) - F(z^* - \epsilon)))} \right) \\
&= (p - z^*) \left(\frac{n^s \kappa^s}{(1 + \kappa^s (1 - F(z^*)))(1 + \kappa^s (1 - F(z^*) + f(z^*)))} + \frac{n^0 \kappa^0}{(1 + \kappa^0 f(z^*))} \right) \qquad (9)
\end{aligned}$$

Profits slightly below the threshold are given by (for $\epsilon \to 0$):

$$\pi(z^* - \epsilon) = (p - (z^* - \epsilon))(\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*-\epsilon)))(1+\kappa^s(1-F(z^*-2\epsilon)))} + \frac{n^0\kappa^0}{(1+\kappa^0(F(z^*)-F(z^*-\epsilon)))(1+\kappa^0(F(z^*)-F(z^*-2\epsilon)))})$$
$$= (p - z^*)(\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*-\epsilon)))^2} + \frac{n^0\kappa^0}{(1+\kappa^0(f(z^*)))^2})$$
$$= (p - z^*)(\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)+f(z^*)))^2} + \frac{n^0\kappa^0}{(1+\kappa^0(f(z^*)))^2})$$

Given proposition (A.I), the data implies that there is a mass point at $z = z^*$ (i.e. that $f(z^*) > 0$). When $f(z^*) > 0$, it holds that $(1+\kappa^j(1-F(z^*)+f(z^*))) > (1+\kappa^j(1-F(z^*)))$ and $(1 + \kappa^0(f(z^*))) > 1$. Therefore, $\pi(z^*) > \pi(z^* - \epsilon)$ and there will be a gap to the left of the threshold.

For the second part of proposition (A.II), define the highest wage offer below the threshold as z'', such that $F(z'') = F(z^* - \epsilon)$. Note that since there is a gap in the earnings distribution below the threshold, if an offer z'' exists in equilibrium, it must be significantly below z^*. In equilibrium any z''-offer must make the same amount of profits as the threshold wage offer z^*.

$$\pi(z'') = (p - z'')\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z'')))(1+\kappa^s(1-F(z''-\epsilon)))} + \frac{n^0\kappa^0}{(1+\kappa^0(F(z^*)-F(z'')))(1+\kappa^0(F(z^*)-F(z''-\epsilon)))}$$
$$= (p - z'')\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)+f(z^*)))^2} + \frac{n^0\kappa^0}{(1+\kappa^0(f(z^*)))^2} \tag{10}$$

Comparing equations (9) and (10) illustrates that $\pi(z'') = \pi(z^*)$ can hold as $\pi(z'')$ increases with decreasing z''. That is, there might be an offer z'' for which $\pi(z'') = \pi(z^*)$ holds.

Using $F(\underline{z}) = 0$, we now determine the lowest wage offer $\underline{z}$ that will be made in equilibrium - if there are any wage offers below z^*.

$$\pi(\underline{z}) = (p - \underline{z})\frac{n^s\kappa^s}{(1+\kappa^s(1-F(\underline{z})))(1+\kappa^s(1-F(\underline{z}-\epsilon)))} + \frac{n^0\kappa^0}{(1+\kappa^0(F(z^*)-F(\underline{z})))(1+\kappa^0(F(z^*)-F(\underline{z}-\epsilon)))}$$
$$= (p - \underline{z})\frac{n^s\kappa^s}{(1+\kappa^s)^2} + \frac{n^0\kappa^0}{(1+\kappa^0(F(z^*)))^2} \tag{11}$$

If $z'' < z^*$ exists and $\underline{z} < z''$, the earnings offer distribution will be continuous between these two values, generating equal profits with the standard trade-off between margins and firm-size. The following section simulates this model and confirms that although type-s

workers have no tax incentive to bunch at z^*, the earnings distributions of both types exhibit a strong mass point under realistic parameter values.

Simulation results

We simulate the stylized version of the model with the following parameter values: $p = 800$ €; $z^* = 325$ €; $\lambda^s = 0.2$; $\lambda^0 = 0.2$; $\delta = 0.1$; $n^s = 1$; $n^0 = 0.1$; $z^r = 0$.

The top-right panel of figure (1) shows a simulated earnings distribution based on the stylized model of the working paper:

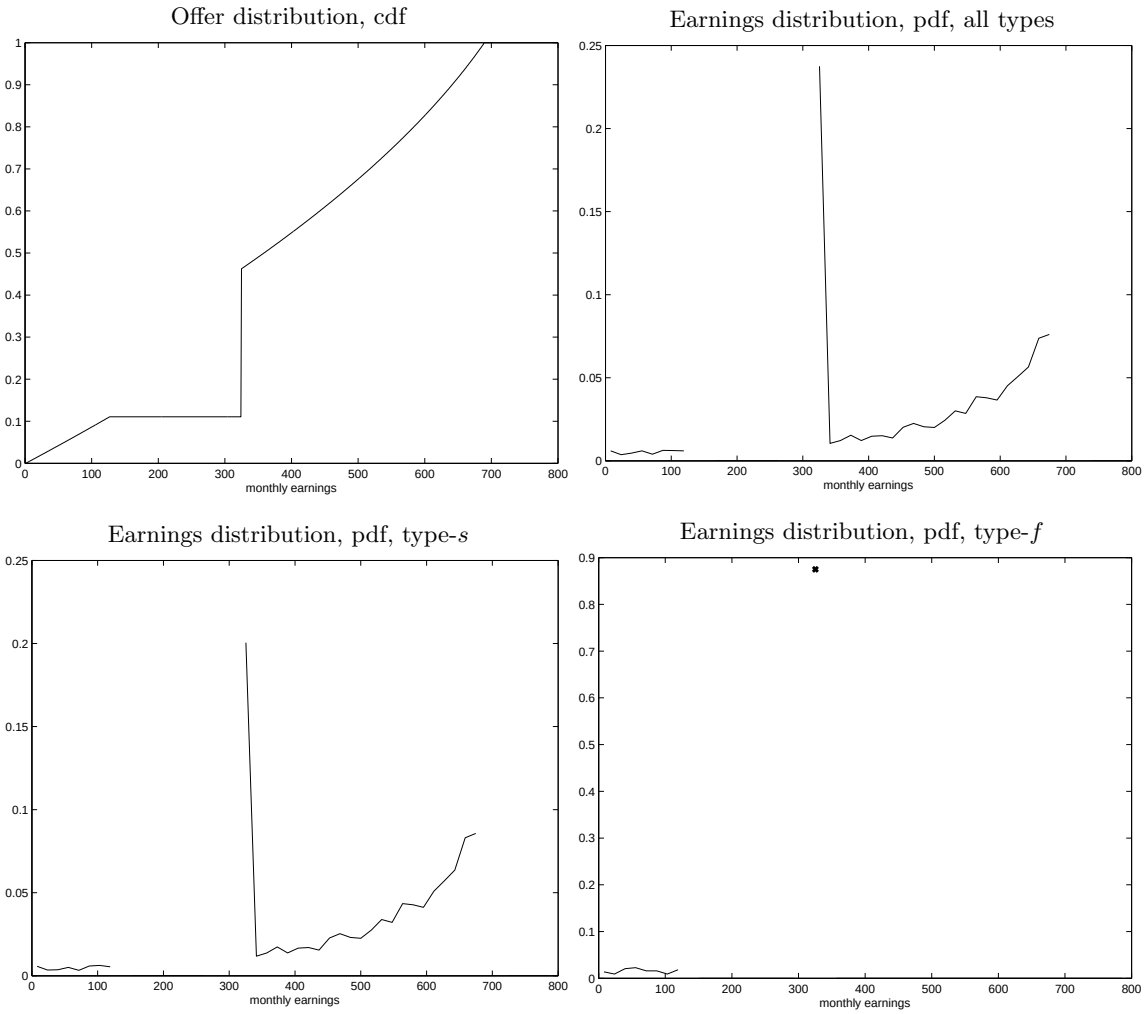
Earnings above the minijob earnings threshold (325 €) increase smoothly on the top-left panel up to a maximum of $\bar{z} = 689$ € in our simulation. At the minijob threshold z^*=325 € there is a large mass point, following proposition (A.I). There is a gap below the threshold in line with proposition (A.II). Firms do not offer any earnings within the interval (127€- 325€). The additional margin of reducing offered earnings does not compensate for the discontinuously lower firm size. The resulting equilibrium cumulative offer distribution the top-left panel of figure (1).

The corresponding earnings distributions of the two types of workers are clearly influenced by the job offer distribution (figure (1)): Although type-s workers have no tax incentive to bunch at z^*, the earnings distributions of both types exhibit a mass point here (for type-f workers the mass point is more prominent and - since these workers accept no job offers above the minijob level - there is no mass above). The upper right panel of figure (1) plots the resulting joint earnings distribution.

B Predicting income tax rates

In Germany, income tax rates depend on household characteristics, but the SIAB data set used in this analysis does not include these. We follow Junge (2017) and implement a two-step procedure. In a first step we use a sophisticated program to calculate taxes in the SOEP based on the most important aspects of the German tax-transfer system. In a second step we impute tax rates in the SIAB dataset based on variables that are available for both data sets.

Figure 1: Offer and earnings distribution by types of workers



Notes: Type-s workers have or seek a small job as second job. Type-f workers have or seek a small job but have no other job. The minijob threshold is at 325€/month. While the cdf is calculated analytically, the pdf-graphs are based on 10000 drawings from the cdf. We simulated the model with the following parameter values: $p = 800$ €; $z^* = 325$ €; $\lambda^s = 0.2$; $\lambda^0 = 0.2$; $\delta = 0.1$; $n^s = 1$; $n^0 = 0.1$; $z^r = 0$.

Tax rates for type-s workers are explained by a Tobit model with first job earnings and sex as explaining variables (table (2)). Based on the same information in the SIAB data we then predict the individual tax rates. We create six discrete income tax groups and allocate type-s and type-f individuals to these. Tax rates are higher for type-s workers as they already have first job earnings. As our sample is restricted to observations with first job earnings of more than 1,000 € (section 4.1 in the main text), all observations exceed the general tax allowance and thus have a strictly positive marginal tax rate. For type-f workers (who often are not liable to pay income tax) this procedure is not very informative (Junge, 2017). Observations in the estimation sample are therefore allocated randomly one of the tax rates based on the distribution of simulated tax rates. Almost

50% of type-f workers have an income tax rate of zero (table (1)). The imputed tax rate for the next 40 % is about eight per cent, for the highest decile it is approximately 20 %.

Table 1: Income tax groups

	type-s				type-f		
Group	%	$\bar{t}^{inc}$	std		%	$\bar{t}^{inc}$	std
1	25	8.32	1.75	4	48.27	0	0
2	65	12.99	4.17	5	41.73	7.92	4.64
3	10	27.6	8.52	6	10	19.47	4.50

Notes: Type-f workers have or seek a small job and have no other job. Type-s workers have or seek a small job as a second job. $\bar{t}^{inc}$ = mean average income tax rate at 326 €- calculated from aggregated observed tax rates for type-f. For type-s, aggregated predicted tax rates based on estimated parameters reported in table (2); std=standard deviation
Data source: SOEP waves 1999-2002

Table 2: Estimation results: tax rates of type-s workers

	coeff.	**s.e.**
Male	-0.0422	0.003
Yearly first job earnings (10k€)	0.0427	0.00075
Constant	0.0354	0.002

Notes: coeff.=regression coefficient, s.e.=standard errors. Data source: SOEP waves 1999-2002

C Predicting hours of work

We do not have precise information on hours of work in our data. We therefore impute working hours in our administrative IAB dataset using survey data from the SOEP. We impute hours based on variables observed in both data sets: sex, gross earnings, industry and education. We split the SOEP estimation sample into two groups of workers. First, employees with only one job and earnings of less than 800 €/month. Second, employees who additionally hold a full-time job paying at least 1,000 €/month and a side job with earnings below 800 €/month. Results show that working hours vary significantly across sectors and educational levels (table (3)). The resulting hours distribution fits the distribution reasonably well, although not all variation is accounted for - see table (4).

Table 3: Estimation results: hours

	coeff.	s.e.
Gross Earnings	0.00	0.00
Sex	0.03	0.05
Sector		
Manufacturing	-0.38	0.11
Energy, Water	-0.28	0.20
Construction	-0.21	0.14
Wholesale and retail	-0.26	0.10
Hotels and restaurants	-0.23	0.15
Transport	-0.40	0.14
Finance	-0.43	0.17
Real Estate	-0.44	0.11
Public Admin	-0.51	0.14
Education	-0.48	0.13
Health	-0.57	0.11
Other Services	-0.28	0.12
Households	-0.35	0.14
Other Sector	-0.33	0.11
Education		
Other Education	0.56	0.13
Basic	0.26	0.09
Middle Voc.	0.28	0.09
Higher Voc.	0.18	0.10
Constant	1.83	0.14

Notes: coeff.=regression coefficient, s.e.=standard errors., dependent variable: weekly hours. Data source: SOEP wave 2001

Table 4: Fit of hours distribution

	Observed	Predicted
mean	57.25	57.14
$p(25)$	23.11	34.45
$p(50)$	43.33	52.17
$p(75)$	86.67	76.91

Notes: $p(x)$: percentile of the hours distribution

D Job offer distribution

We calculate the earnings offer distribution by solving the profit maximization problem successively for different firms in the market.[1] The same strategic arguments outlined in section A apply here: Firms wish to pay the lowest possible wages, but if there is a point mass in the utility distribution implied by other firms' job offers, firms have an incentive to offer slightly higher wages. The algorithm reproduces the outcome of this non-cooperative game and converges to the equilibrium job offer distribution.

We implement the following algorithm:

(i) All firms simultaneously draw from the exogenous hours distribution.

(ii) We discretize the wage space using 100 wage points. 99 of these are equally spaced on the support between b and p. We also ensure that posting a wage that results in earnings *precisely* at the minijob tax exemption threshold z^* is an option for every hours draw.

(iii) Start of the iterative algorithm. Firm $j \in 1, ..., J$ is allowed to update their wage offer w_j.

(iv) Firm j calculates profit levels associated with different choices of wage offer w_j. To do this:

- Calculate $F^v_{-j}(v(w,h))$, the distribution of instantaneous utility (to workers) implied by all other job offers (their combination of hours and wages).
- Given the distribution of instantaneous utility implied by these offers, calculate expected firm size based on the number of workers of type s and f an offer is expected to attract $l(v(w,h)) = l^s(v(w,h)) + l^f(v(w,h))$.
- Profits are then calculated as:

$$\pi(w) = \begin{cases} (p-w)\ h(\frac{n^s\kappa^s}{(1+\kappa^s\ (1-F^v(v(w,h))))^2} + \frac{(1-\theta)n^f\kappa^f}{\left(1+\kappa^f(1-F^v(v(w,h)))\right)^2} + \frac{\theta n^f\kappa^f}{\left(1+\kappa^f(1-F^v(v(w,h)|z\leq z^*))\right)^2}) & \forall z \leq z^* \\ (p-w)\ h(\frac{n^s\ \kappa^s}{(1+\kappa^s(1-F^v(v(w,h))))^2} + \frac{(1-\theta)n^f\kappa^f}{\left(1+\kappa^0(1-F^v(v(w,h)))\right)^2}) & \forall z > z^* \end{cases}. \tag{12}$$

(v) Choose the wage offer w_j that maximizes profits.

[1] If firms did not have different hours requirements, the offer distribution could be characterized by a system of equal profit conditions. If hours requirements vary, however, they may make different profits in equilibrium.

(vi) update $F^v(v(w,h))$ and move back to point (iii)

Repeating the algorithm allows firms to respond to other firms' best responses etc. We find that after 1,000 iterations, the distribution is fairly stable: The estimated fraction of offers with earnings below or at the threshold, for example, has a standard deviation of less than 0.02 when the algorithm is repeated ten times. To increase precision, we take the average offer distribution of ten repetitions of the algorithm. The gains of increasing the number of iterations or wage grid points are small relative to the computing cost.

E Maximum likelihood grid-search

We determine the values of θ, α and σ that maximize the likelihood function using a two-step grid search procedure (Hansen, 2016). Gradient-based approaches are inappropriate because the costly calculation of the job offer distribution has to be repeated every iteration. It further contains discontinuous elements due to the tax system that render the likelihood function potentially non-smooth. For θ and α we evaluate the likelihood in the first step at 11 equally spaced grid points in the interval $[0, 1]$. In the second step the grid is narrowed down to steps of 0.02. For σ we use a grid of 10, 50, 100, 150 and 200 as grid points in the first step. The grid is narrowed down to steps of 20 in the second step.

F Robustness test: labor demand reactions

An important aspect of the minijob tax exemption concerns workers' participation in the labor market. We model these extensive margin reactions by the estimated fraction of workers who no longer participate in the market when the tax exemption is removed. If we assume that other frictional parameters remain unchanged, the overall number of vacancies posted by firms must decrease: Workers whose jobs have become non-competitive when taxed are not replaced by other workers. This may be reasonable for certain low-productivity jobs but less so in other contexts. We here perform a robustness test in this respect.

We now assume that the minijobs that were destroyed are replaced by other jobs. Thus the number of vacancies per period posted by firms stays constant. This implies that job offer arrival rates for type-s and fa workers must increase. Keeping the ratio between both constant, this implies $\lambda^s = 0.0513$ and $\lambda^0 = 0.3908$. This is comparable to introducing a matching function, accounting for on-the-job search and differing arrival

rates across worker groups.

The remaining workers in the market substantially increase their net earnings and utility - despite some workers now losing their tax exemption. While total hours decrease by about 12%, this is overcompensated by an increase in the average wage rate, such that total gross earnings increase. Tax revenues increase accordingly. In practice, reactions are presumably somewhat more muted. For example, if the economic incidence of the tax exemption does not entirely fall on employees, firms profit from the lower wage rates for minijobs. Removing the tax exemption might then lead to a decrease in vacancies. The differences between both scenarios imply that labor demand responses are important for comprehensively evaluating the minijob regulation.

Table 5: Removing the tax and SSC exemption - upper bound scenario

Change in	**Total**	type-s	type-fa	type-fo
mean hourly wages $\bar{w}$ (€)	**0.78**	0.09	0.92	.
mean weekly hours of work $\bar{h}$	**18.05**	4.98	9.92	.
total hours of work $\sum_i^N h_i$ (%)	**-10.01**	41.27	15.15	-100
mean monthly gross earnings $\bar{z}$ (€)	**179.98**	36.67	156.08	.
total earnings $\sum_i^N z_i$ (%)	**9.72**	47.45	41.38	-100
mean net earnings $\bar{c}$ (€)	**83.96**	24.98	70.31	.
mean utility $\bar{v}$	**69.11**	20.25	55.81	-199.34
jobs (1000s)	**-1360.5**	163	120.5	-1644
total taxes (million €)	**341.04**	39.64	387.61	-86.21

Notes: $\bar{w}$, $\bar{h}$, $\bar{z}$, and $\bar{c}$ conditional on employment; utility $\bar{v}$ not conditional on employment; Estimates of changes in number of jobs and tax revenues based on extrapolations from the sample to the population; Type-s workers have or seek a small job as a second job. Type-fa workers have or seek a small job, have no other job and would accept any small job. Type-fo workers have or seek a small job, have no other job and accept only minijobs.

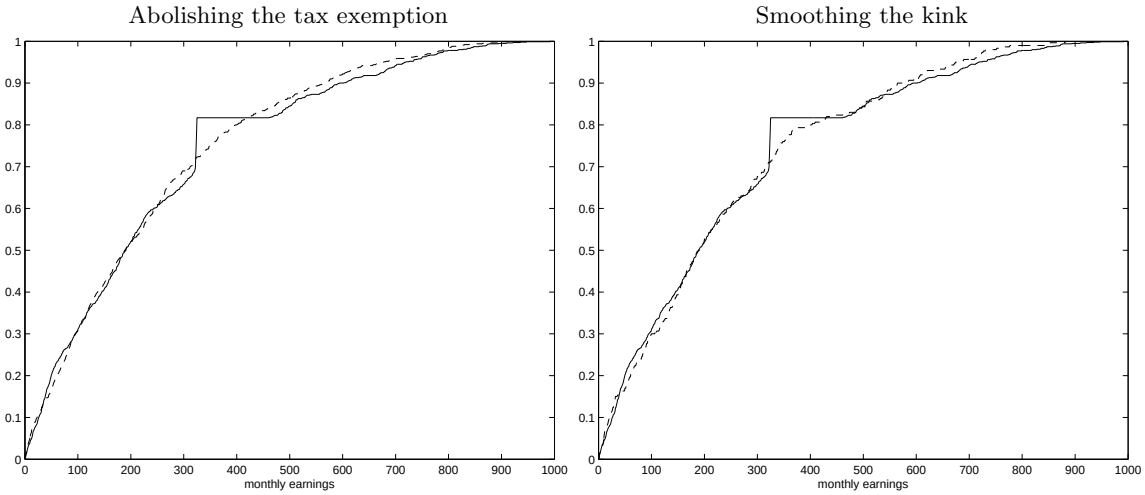
G Small job market with one worker type

When we simulate removing the minijob tax exemption, two effects operate: First, one group of the workforce chooses no longer to participate. Since this group is large relative to the market, equilibrium effects for other workers are significant. Second, incentives for the remaining labor supply change due to the new tax schedule. Firms anticipate this and adjust their wage offers accordingly. This section separates the two effects: Imagine all workers in the market for small jobs are of type fa. That is, all workers are eligible for a potential tax exemption but would work when subject to full taxes. Counterfactual simulations in such a market are insightful because they abstract from compositional changes in the workforce: All workers remain in the market after the tax exemption is abolished.

When the tax exemption is removed, firms that offered jobs at the threshold in the status quo now mostly offer jobs with higher earnings (left panel of figure (2)). This is not surprising as jobs with high earnings are relatively more attractive now. However, some firms also reduce their earnings offer. The distribution of earnings below the threshold is not affected. Removing the tax exemption has a positive effect on average (and total) gross earnings (table (6)). Not only do workers receive such offers more frequently, workers are also more likely to accept higher earnings offers.

An alternative is to smooth the minijob discontinuity by gradually increasing the tax rate for earnings between 325 and 800 € (see section 6.2 in the main text). The right column of table (6) shows the effect of smoothing the tax schedule in a market with only type-fa workers: Firms that offered jobs at the threshold in the status quo increase their earnings offers. Relative to the status quo, the distribution above the threshold is more compressed. Firms react to the change in workers' incentives due to the high implicit marginal tax rate introduced by the reform.

Figure 2: Cumulative earnings offer distribution - status quo vs. reform - only type fa workers



Notes: Here only type-fa workers - these have or seek a small job, have no other job and would accept any small job. The minijob threshold is at 325€/month. The solid line represents the status quo, the dashed line the counterfactual.

H Alternative definition of the job-seeking population

Classifying individuals as job seeking in the main text relies on several assumptions. Workers need to fulfill two conditions to be classified as second jobbers. First, the respective individual needs to be employed with earnings exceeding 1,000 € per month. Second, the individual must have had at least one small job as a second job within the analysis

Table 6: Effects of removing and smoothing the minijob tax exemption - only type-fa workers

Change in	Removal	Smoothing
mean hourly wages $\bar{w}$ (€)	0.50	0.33
mean weekly hours of work $\bar{h}$	-0.56	-2.60
total hours of work $\sum_i^N h_i$ (%)	-0.58	-2.71
mean monthly gross earnings $\bar{z}$ (€)	40.73	6.93
total earnings $\sum_i^N z_i$ (%)	8.67	1.48
mean net earnings $\bar{c}$ (€)	-10.56	25.28
mean utility $\bar{v}$	-7.67	17.43
jobs (1000s)	0.00	0.00
total taxes (million €)	269.64	-76.41

Notes: Removal: Complete removal of tax exemption; Smoothing: Minijob earnings notch replaced by a kink; taxes in million €; $\bar{w}$, $\bar{h}$, $\bar{z}$, and $\bar{c}$ conditional on employment; utility $\bar{v}$ not conditional on employment; Estimates of changes in number of jobs and tax revenues based on extrapolations from the sample to the population; Type-fa workers have or seek a small job, have no other job and would accept any small job.

period (April 1999 to February 2002). On the one hand, we disregard individuals seeking a second job who have not been able to find one in the analysis period. On the other hand, we might misclassify some individuals as seeking a second job who have stopped searching. The large number of job-seeking type-s workers suggests the latter may be particularly important. In this section we test a tighter definition of job-seekers in the market for second jobs: Only individuals who find a small job as second job spell in the observation window are included. The number of job-seeking second jobber spells decreases by 10,000 to less than 40,000 (table 7). Similarly, we classify formal unemployment spells as type-f when the respective individuals have had a small job spell in the sample period. Not all small job spells, however, are preceded by a formal unemployment spell. In this section we additionally classify all those individuals as small-job seeking who are out of the labor force. Spells of non-participation are specified as the gap between two observed spells. The amount of type-f job-seekers roughly doubles to 116,182 (table (7)).

Table 7: Total number of spells by type and employment - alternative definition of market

	type-s	type-f
Job-seeking	38,048	116,182
Employed in small job	30,121	287,255

Notes: Type-f workers have or seek a small job and have no other job. Type-s workers have or seek a small job as a second job. Data source: SIAB

As expected, the offer arrival rate of type-f workers is now found to be lower than in the benchmark results. By contrast, the offer arrival rate of type-s workers is almost constant. The estimated value of θ moves from 0.37 to 0.20. There is presumably a significant number

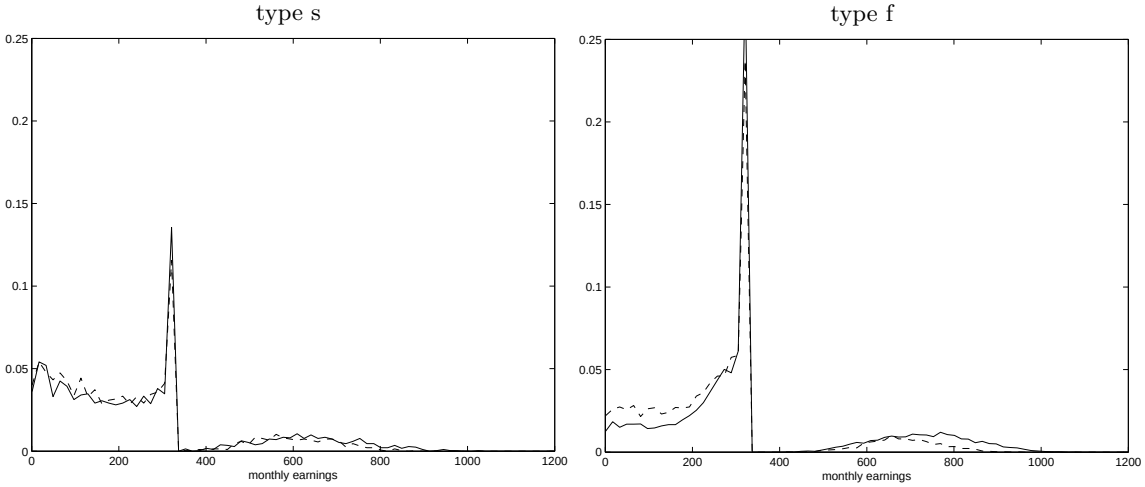
of people who irregularly use minijobs, and fall under one the definition of job-seeking but not another. Reassuringly though, the predicted realized earnings distributions are very similar (figure (3)). For type-f workers there is slightly more mass to the left of the cap and slightly less at and above the threshold.

Table 8: Parameter estimates - alternative definition of market

	point estimate	s.e.
δ^s	0.0474	0.0005
δ^f	0.0451	0.0002
λ^s	0.0296	0.0002
λ^f	0.1571	0.0028
θ	0.2064	0.0775
α	0.8843	0.1121
σ	67.500	6.963

Notes: s.e. - bootstrapped standard deviation, λ^j – arrival rate for worker type j, δ^j – job destruction rate of worker type j, α – relative weight of consumption and leisure in the utility function, θ – fraction of type-f workers who do not accept jobs with earnings exceeding the threshold, σ – standard deviation of measurement error

Figure 3: Predicted earnings distribution for alternative job-seeking populations



Notes: Type-f workers have or seek a small job and have no other job. Type-s workers have or seek a small job as a second job. The minijob threshold is at 325€/month. Solid curve pertains to main specification, dashed curve to alternative definition of the job-seeking population. Data source: SIAB.

I Discrete variation in working hours

In the main text, we assume continuous variation in hours worked. We here discuss the model equilibrium with discrete variation in hours: Weekly hours worked are in one of two categories h_k, such that $k \in (1, 2)$. In the market for low-paid jobs, this may correspond to 10 and 20 hours of working. We later consider how this model generalizes to three or

more hours categories.

Firms set wage rates, w, workers derive utility from consumption and leisure. In order to simplify notation, we follow Shephard (2017) and define $q_2(w) = w$ and $U(q_1(w), h_1) = U(q_2(w), h_2) = U(w, h_2)$, so $q_1(w)$ is a function that denotes the wage rate that makes individuals indifferent between working with few ($k = 1$) hours at $q_1(w)$ or working more ($k = 2$) hours at w. Depending on preferences, individuals may require a low-hours wage premium or accept a low-hours wage penalty.

Worker mobility

The flow into and out of small jobs must balance (see section A). The flow of workers of type $j \in (s, 0a, 0m)$ from and into jobs with hours h_k and wage rate w is

$$D^j(w)g_k^j(q_k(w))e_k^j = \lambda^j f_k(q_k(w)) \left(u^j + G_1^j(q_1(w) - \epsilon)e_1^j + G_2^j(w - \epsilon)e_2^j\right) \tag{13}$$

with $D^j(w) = [\delta + \lambda^j((1 - F_2(w)) + (1 - F_1(q_1(w))))]$ for $j \in (s, 0a)$. Equation (14) states the corresponding definition for workers of type $0m$ who do not accept jobs with wage rates larger than w_k^*.

$$D^{0m}(w) = \begin{cases} [\delta^0 + \lambda^0((F_2(w_2^*) - F_2(w)) + (F_1(w_1^*) - F_1(q_1(w))))] & \forall w \leq w_2^* \text{ and } q_1(w) \leq w_1^* \\ [\delta^0 + \lambda^0(F_2(w_2^*) - F_2(w))] & \forall w \leq w_2^* \text{ and } q_1(w) > w_1^* \end{cases} \tag{14}$$

The LHS of equation (13) pertains to workers who leave a job in a sector k with wage $q_k(w)$. For $k = 2$ this group consists of workers who move from sector 2 to sector 1 ($\lambda^j(1 - F_1(q_1(w)))g_2^j(w)e_2^j$ for $j \in (s, 0a)$), who move to a better paying job within sector 2 ($\lambda^j(1 - F_2(w))g_2^j(w)e_2^j$ for $j \in (s, 0a)$) and who lose their small job ($\delta^j g_2^j(w)e_2^j$). The RHS pertains to workers who start a job in sector k with wage rate $q_k(w)$. For $k = 2$ this consists of workers who move from sector 1 to sector 2 ($\lambda^j f_2(w)G_1^j(q_1(w) - \epsilon)e_1^j$), who changes jobs within sector 2 ($\lambda^j f_2(w)G_2^j(w - \epsilon)e_2^j$) and who were previously job-seeking ($\lambda^j f_2(w)$). The overall flow (i.e. both sectors) due to separations from jobs with wage rate of no greater than w is:

$$\begin{aligned} &(G_1^j(q_1(w))e_1^j + G_2^j(w)e_2^j)D^j(w) = \lambda^j u^j F_1(q_1(w)) + \lambda^j u^j F_2(w) \\ &= \lambda^j u^j + \lambda^j u^j - \lambda^j u^j(1 - F_1(q_1(w))) - \lambda^j u^j(1 - F_2(w)) \end{aligned} \tag{15}$$

Recall that flows of workers entering and exiting small jobs must be equally large in

equilibrium, i.e.

$$\delta^j(n^j - u^j) = \begin{cases} \lambda^j u^j & \text{for } j = s \\ \lambda^j u^j F(z^*) & \text{for } j = f \end{cases} \tag{16}$$

We then have

$$G_1^j(q_1(w))e_1^j + G_2^j(w)e_2^j = \frac{\delta^j n^j - u^j D^j(w)}{D^j(w)}. \tag{17}$$

By combining equations (13) and (17) we obtain

$$g_k^j(q_k(w))e_k^j = \frac{\lambda^j f_k(q_k(w)) \left[u^j + \frac{\delta^j n^j - u^j(D^j(w-\epsilon))}{D^j(w-\epsilon)}\right]}{D^j(w)} \tag{18}$$

Firm size

The number of workers of type j in steady-state employed at a firm in sector k which offers wage rate $q_k(w)$ is

$$\begin{aligned} l_k^j(q_k(w)) &= \frac{g_k^j(q_k(w))e_k^j}{f_k(q_k(w))} \\ &= \frac{\lambda^j \delta n^j}{D^j(w)D^j(w-\epsilon)}. \end{aligned} \tag{19}$$

The steady state firm size is then

$$\begin{aligned} l_k(q_k(w)) &= l_k^s(q_k(w)) + l_k^{0a}(q_k(w)) + l_k^{0m}(q_k(w)) \\ &= \begin{cases} \frac{\lambda^s \delta^s n^s}{D^s(w)D^s(w-\epsilon)} + \frac{\lambda^0 \delta^0 n^{0a}}{D^{0a}(w)D^{0a}(w-\epsilon)} + \frac{\lambda^0 \delta^0 n^{0m}}{D^{0m}(w)D^{0m}(w-\epsilon)} & \forall w \le w^* \\ \frac{\lambda^s \delta^s n^s}{D^s(w)D^s(w-\epsilon)} + \frac{\lambda^0 \delta^0 n^{0a}}{D^{0a}(w)D^{0a}(w-\epsilon)} & \forall w > w^* \end{cases}. \end{aligned} \tag{20}$$

Following the standard arguments of profit equalization, we find the following (the reasoning is parallel to the case without hours variation):

Proposition (I.1) *There can be (at most one) mass point in the wage offer distribution at the threshold in each sector, i.e. at wages* $w_k^* \equiv \frac{z^*}{h_k}$.

Sketch of Proof: The following argument closely mirrors the argument in the case of homogeneous hours. We compare profits at the threshold value with profits above. We find that if there exist offers above, there must be a mass point at the threshold.

The profit of a sector k firm offering wage rate $q_k(w)$ can be expressed as $\pi_k(q_k(w)) = (ph_k - q_k(w)h_k)l_k(q_k(w))$. We first state the profits of a type-2-firm, assuming that $q_1(w_2^*) \le w_1^*$.

$$\begin{aligned}\pi_2(w_2^*) &= \frac{\lambda^s\delta^s n^s}{D^s(w_2^*)D^s(w_2^*-\epsilon)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(w_2^*)D^{0a}(w_2^*-\epsilon)} + \frac{\lambda^0\delta^0 n^{0m}}{D^{0m}(w_2^*)D^{0m}(w_2^*-\epsilon)} \\ &= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(w_2^*))+(1-F_1(q_1(w_2^*))))][\delta^s+\lambda^s((1-F_2(w_2^*-\epsilon))+(1-F_1(q_1(w_2^*)-\epsilon)))]} + \\ &+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(w_2^*))+(1-F_1(q_1(w_2^*))))][\delta^0+\lambda^0((1-F_2(w_2^*-\epsilon))+(1-F_1(q_1(w_2^*)-\epsilon)))]} + \\ &+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0(F_1(w_1^*)-F_1(q_1(w_2^*)))][\delta^0+\lambda^0(f_2(w_2^*)+(F_1(w_1^*)-F_1(q_1(w_2^*)-\epsilon)))]} \end{aligned} \tag{21}$$

Evaluated marginally above the threshold, profits are

$$\begin{aligned}\pi_2(w_2^*+\epsilon) &= \frac{\lambda^s\delta^s n^s}{D^s(w_2^*+\epsilon)D^s(w_2^*)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(w_2^*+\epsilon)D^{0a}(w_2^*)} \\ &= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(w_2^*+\epsilon))+(1-F_1(q_1(w_2^*+\epsilon))))][\delta^s+\lambda^s((1-F_2(w_2^*))+(1-F_1(q_1(w_2^*))))]} + \\ &+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(w_2^*+\epsilon))+(1-F_1(q_1(w_2^*+\epsilon))))][\delta^0+\lambda^0((1-F_2(w_2^*))+(1-F_1(q_1(w_2^*))))]} \end{aligned} \tag{22}$$

Equations (21) and (22) show that the equal profit condition can only hold if there is a mass point in the offer distribution of sector 2 at w_2^*. By symmetry, note that the same argument can be made with respect to a type-1 firm. However, if the utility of a threshold offer lies in the "gap area" due to a threshold in another sector, it may be the case that there is no mass point in that sector. This explains the restriction "at most one" in Proposition (I.1) and completes our discussion.

We now consider the influence of thresholds in other hours sectors on the wage distribution. Consider a firm of type 2, i.e. seeking a worker to work for h_2 hours. The impact of a potential mass point in the offer distribution of sector 1 at w_1^* depends on the relation between w_2^*, $q_1(w_2^*)$ and w_1^*.

Proposition (I.2) *There will be no wage offers at wage levels (and in a certain interval below this level) that offer the same utility as is available at threshold wages $w^*_{j\neq\ k}$ in other sectors.*

The intuition for Proposition (I.2) is the following: It is a dominated strategy to offer a wage rate that is equal in utility to an offer made by several other firms. A slightly higher offer will attract all workers from these firms at only marginal cost. By Proposition (I.1), wage offers at earnings thresholds generate mass points in the wage offer distributions. Thus for example a type-2 firm will offer a wage rate slightly larger than $\tilde{w}_2$ (where

$U(\tilde{w}_2, h_2) = U(w_1^*, h_1).)$ in order to additionally attracts workers from this positive mass of sector 1 firms. This implies that there must be a gap in the wage offer distribution at $\tilde{w}_2$. How much below this utility value an offer can be sustained in equilibrium will depend on the parameters of the model in an analogous way to the potential existence of offers below the threshold offer in the homogeneous case.

Sketch of proof: Let $\tilde{w}_2$ denote the wage rate which satisfies $U(\tilde{w}_2, h_2) = U(w_1^*, h_1)$. If $\tilde{w}_2 > w_2^*$ the profits of a sector 2 firm offering wage rate $\tilde{w}_2$ and slightly above are:

$$\begin{aligned}
\pi_2(\tilde{w}_2) &= \frac{\lambda^s \delta n^s}{D^s(\tilde{w}_2) D^s(\tilde{w}_2 - \epsilon)} + \frac{\lambda^0 \delta^0 n^{0a}}{D^{0a}(\tilde{w}_2) D^{0a}(\tilde{w}_2 - \epsilon)} + \frac{\lambda^0 \delta^0 n^{0m}}{D^{0m}(\tilde{w}_2) D^{0m}(\tilde{w}_2 - \epsilon)} \\
&= \frac{\lambda^s \delta^s n^s}{[\delta^s + \lambda^s((1 - F_2(\tilde{w}_2)) + (1 - F_1(w_1^*)))][\delta^s + \lambda^s((1 - F_2(\tilde{w}_2 - \epsilon)) + (1 - F_1(w_1^* - \epsilon)))]} + \\
&+ \frac{\lambda^0 \delta^0 n^{0a}}{[\delta^0 + \lambda^0((1 - F_2(\tilde{w}_2)) + (1 - F_1(w_1^*)))][\delta^0 + \lambda^0((1 - F_2(\tilde{w}_2 - \epsilon)) + (1 - F_1(w_1^* - \epsilon)))]} + \\
&+ \frac{\lambda^0 \delta^0 n^{0m}}{[\delta^0 + \lambda^0((F_2(w_2^*) - F_2(\tilde{w}_2)) + (F_1(w_1^*) - F_2(w_1^*))))][\delta^0 + \lambda^0((F_2(w_2^*) - F_2(\tilde{w}_2 - \epsilon)) + (F_1(w_1^*) - F_2(w_1^* - \epsilon)))]} \\
&= \frac{\lambda^s \delta^s n^s}{[\delta^s + \lambda^s((1 - F_2(\tilde{w}_2)) + (1 - F_1(w_1^*)))][\delta^s + \lambda^s((1 - F_2(\tilde{w}_2)) + (1 - F_1(w_1^*) + f_1(w_1^*))))]} + \\
&+ \frac{\lambda^0 \delta^0 n^{0a}}{[\delta^0 + \lambda^0((1 - F_2(\tilde{w}_2)) + (1 - F_1(w_1^*)))][\delta^0 + \lambda^0((1 - F_2(\tilde{w}_2)) + (1 - F_1(w_1^*) + f_1(w_1^*))))]} + \\
&+ \frac{\lambda^0 \delta^0 n^{0m}}{[\delta^0 + \lambda^0((F_2(w_2^*) - F_2(\tilde{w}_2)))][\delta^0 + \lambda^0((F_2(w_2^*) - F_2(\tilde{w}_2)) + f_1(w_1^*))]}
\end{aligned} \tag{23}$$

$$\begin{aligned}
\pi_2(\tilde{w}_2 - \epsilon) &= \frac{\lambda^s \delta^s n^s}{D^s(\tilde{w}_k - \epsilon) D^s(\tilde{w}_k - 2\epsilon)} + \frac{\lambda^0 \delta^0 n^{0a}}{D^{0a}(\tilde{w}_2 - \epsilon) D^{0a}(\tilde{w}_2 - 2\epsilon)} + \frac{\lambda^0 \delta^0 n^{0m}}{D^{0m}(\tilde{w}_2 - \epsilon) D^{0m}(\tilde{w}_2 - 2\epsilon)} \\
&= \frac{\lambda^s \delta^s n^s}{[\delta^s + \lambda^s((1 - F_2(\tilde{w}_2 - \epsilon)) + (1 - F_1(w_1^* - \epsilon)))][\delta^s + \lambda^s((1 - F_2(\tilde{w}_2 - 2\epsilon)) + (1 - F_1(w_1^* - 2\epsilon)))]} + \\
&+ \frac{\lambda^0 \delta^0 n^{0a}}{[\delta^0 + \lambda^0((1 - F_2(\tilde{w}_2 - \epsilon)) + (1 - F_1(w_1^* - \epsilon)))][\delta^0 + \lambda^0((1 - F_2(\tilde{w}_2 - 2\epsilon)) + (1 - F_1(w_1^* - 2\epsilon)))]} + \\
&+ \frac{\lambda^0 \delta^0 n^{0m}}{[\delta^0 + \lambda^0((F_2(w_2^*) - F_2(\tilde{w}_2 - \epsilon)) + (F_1(w_1^*) - F_2(w_1^* - \epsilon)))][\delta^0 + \lambda^0((F_2(w_2^*) - F_2(\tilde{w}_2 - 2\epsilon)) + (F_1(w_1^*) - F_2(w_1^* - 2\epsilon)))]} \\
&= \frac{\lambda^s \delta^s n^s}{[\delta^s + \lambda^s((1 - F_2(\tilde{w}_2)) + (1 - F_1(w_1^*) + f_1(w_1^*)))]^2} + \\
&+ \frac{\lambda^0 \delta^0 n^{0a}}{[\delta^0 + \lambda^0((1 - F_2(\tilde{w}_2)) + (1 - F_1(w_1^*) + f_1(w_1^*)))]^2} + \\
&+ \frac{\lambda^0 \delta^0 n^{0m}}{[\delta^0 + \lambda^0((F_2(w_2^*) - F_2(\tilde{w}_2)) + f_1(w_1^*))]^2}
\end{aligned} \tag{24}$$

$$
\begin{aligned}
\pi_2(\tilde{w}_2+\epsilon) &= \frac{\lambda^s\delta^s n^s}{D^s(\tilde{w}_2+\epsilon)D^s(\tilde{w}_2)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(\tilde{w}_2+\epsilon)D^{0a}(\tilde{w}_2)} + \frac{\lambda^0\delta^0 n^{0m}}{D^{0m}(\tilde{w}_2+\epsilon)D^{0m}(\tilde{w}_2)} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2+\epsilon))+(1-F_1(w_1^*+\epsilon)))][\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]} + \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2+\epsilon))+(1-F_1(w_1^*+\epsilon)))][\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]} + \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2+\epsilon)))][\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2)))]} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]^2} + \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]^2} + \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2)))]^2}
\end{aligned}
\tag{25}
$$

As $f_1(w_1^*) > 0$ and $\epsilon \to 0$, it holds that $\pi_2(\tilde{w}_2-\epsilon) < \pi_2(\tilde{w}_2) < \pi_2(\tilde{w}_2+\epsilon)$. This implies that there will be no wage offers of value $\tilde{w}_2$. As $(ph-wh)$ increases with decreasing w, there might be a wage rate w' where it holds that $\pi_2(w') = \pi_2(\tilde{w}_2+\epsilon)$. This implies that $f_2(.)$ exhibits a gap in the interval (w',$\tilde{w}_2$). If $\tilde{w}_2 < w_2^*$, the terms in equations (23) and (25) referring to workers of type $0m$ drop out. Although this might reduce the extent of the gap, $\pi_2(\tilde{w}_2) < \pi_2(\tilde{w}_2+\epsilon)$ still holds. If $\tilde{w}_2 = w_2^*$ the necessary size of the mass point at w_2^* to balance the loss of type-$0m$ workers decreases (in comparison to $\tilde{w}_2 \neq w_2^*$). How large the gap is, i.e. whether any offers will be made below $\tilde{w}_2$ will depend on the economic environment captured by the parameters of the model.

Part II

Questions & Answers

Contents

J Modeling firms' wage-setting in the small jobs market

J.1 What if we allow firms to offer both full-time and small jobs?

We do not require firms active in the market for small jobs to refrain from offering full-time jobs. Firms might offer both full-time and small jobs and be active in both markets. As long as there is no interaction between these two markets, this is fine - analogously to second-jobbers who are active in both full-time and small jobs markets, firms may also be. However, we refrain from modeling the market for full-time jobs. An example of an interaction that we do not allow for would be if firms turn a small job into a full-time job or vice-versa as a result of the changes in the labor market policy options we analyze.

We first sketch three very simple approaches for thinking about why and how firms may offer both full-time and small jobs, viz. how they may choose in which market to operate. These models make sense of the role that hours requirements play in our model. Section J.1.4 considers empirical evidence for such hours restrictions. In section J.1.5 we then discuss the implications of these models for our analysis of counterfactual policies. They may therefore help us think in a structured way about the labor demand effects of changing the minijob tax exemption.

J.1.1 Why offer full-time jobs?

Any model would have to start from the following observation: There is a rationale for both firms and workers to try to benefit from the lower rates of labor taxes offered by minijobs: Put simply, if the government takes less, there is more surplus for workers and firms to share. Why wouldn't firms *only* offer minijobs?[2]

The instinctive response "this only works for low-paid jobs" is only true in the sense of earnings. Very high wage rates are entirely possible, as the earnings threshold contains no limitations on the (hourly) wage rate. However, higher wage rates translate to fewer hours given the earnings threshold. This thus leads us to believe that herein must lie the key constraint: Firms have certain production technologies that make certain weekly working hours more attractive than others - and these technologies may differ across firms. We go through two models that achieve this by fixed costs (of hiring workers) and one simply by asserting productivity differentials.

[2]In fact, in some sectors this is nearly the case: In hospitality or retail, the proportion of minijobbers is indeed very high, with only managers sometimes not on minijob contracts.

J.1.2 Two models of fixed costs of employment

Assume firms face fixed costs per worker. The cause of these could be very material: personal clothing, protective gear, laptop, office space, software license etc. Such fixed costs would lead to a preference by firms, ceteris paribus, of increasing working hours.[3] In our setting, the benefits of spreading fixed costs across more hours can be off-set by incentives emanating from the budget constraint via the minijob tax exemption.

Fixed costs with (ex-ante) identical firms

Assume that productivity per hour is constant and fixed costs are homogeneous across firms. Recall firms' profits are given as $\pi = [p - w]\ h\ l(v(w,h))$ where p is hourly productivity, $w = \frac{z}{h}$ is the gross hourly wage rate and $l(v(w,h))$ the size of the labor force of a firm offering jobs with utility $v(w,h)$, with each firm offering one type of job.

Profit-maximizing firms now face a two-dimensional problem of choosing hours and wages. The equilibrium in the wage-posting game can be expected to follow the lines of Burdett and Mortensen (1998): In their framework "distance from b" can be interpreted as the price that firms must pay in the wage-space to increase their workforce at the cost of lower margins. In the hours-space, it will now be "distance from maximum hours", since with constant productivity and fixed costs per worker, the maximum (e.g. legally binding) number of hours per worker will lead to lowest hourly labor costs. Firms will again choose different levels of attractiveness and the trade-off between making more margin per worker versus having a larger workforce applies again. Having the extra strategic space of choosing hours would only change this dynamic a little. Profit-maximizing firms can choose different strategies in equilibrium given that compensating differences in firm size and profits per worker lead to the same expected profits for different firm strategies.

In combination with the incentives emanating from the tax exemption threshold, it should be possible to account for an equilibrium with a large number of firms requiring workers to work full time and a large number benefiting from the minijob tax exemption - and some mass in between.

[3] Note that there is a literature on fixed costs of work on the labor supply side going back at least to Cogan (1981). Recently, Erosa et al. (2016) stress the importance of this factor for aggregate labor supply outcomes for example. Without having gone through this formally, we suspect that the effects on equilibrium hours distribution of fixed costs on the side of workers - e.g. travel costs related to commuting - will be similar to firm-side fixed costs in equilibrium.

In conclusion, this set-up is elegant in the sense of producing significant variation in job offers with minimal ingredients. It provides an alternative justification for firms to post jobs with different weekly hours without reference to exogenous factors determining "hours requirements". By contrast, it will probably also face the empirical difficulties that the classic Burdett-Mortensen model faces, as it should also predict an increasing utility-distribution, which translates to a poor fit of the data. Note also that in this model all firms are equally competitive and profitable. A random set of firms will offer minijobs (or in degenerate settings, no or all firms).

Variation in fixed costs across firms

The solution concept in our submission resembles Bontemps et al. (1999) in the sense that some firms find it easier to attract workers than others. In their study, ex-ante productivity differentials across firms play this role. In our paper, hours requirements play this role. Some hours requirements are unfortunate given workers' preferences and the tax schedule. This breaks the curse of producing an increasing (utility) offer distribution.

Our solution of simply assuming exogenous hours requirements may appear too restrictive. It would be possible to set the level of firm heterogeneity one level "deeper", for example using heterogeneity in fixed costs. Assume the model as in our paper, but every firm j is now allowed to choose the working hours of their posted job offers while facing a firm-specific fixed cost γ_j for every employed worker that accrues in every period independent of weekly working hours. Labor costs per worker, $\Gamma_j(.)$ are then given by

$$\Gamma_j(w,h) = w_j \ h_j + \gamma_j. \tag{26}$$

The firm's problem in deciding the characteristics of a wage posting is then to maximize profits, given by

$$\max_{h,w} \pi = [p \ h_j - \Gamma_j(w,h)] \ \ l_j(v(w,h)) \tag{27}$$

$$\max_{h,w} \pi = [(p - w_j) \ h_j - \gamma_j] \ \ l_j(v(w,h)). \tag{28}$$

Note that in line with the model of the labor market presented in our paper, the size of the available labor force is the result of stochastic search processes in the labor market and depends on the (instantaneous) utility v of a job offer with characteristics w, h as well

as the size of different groups of workers (types s,fo,fa).

$$\max_h \pi = [(p - w_j) \;\; h_j - \gamma_j] \;\; l_j \left(v(w,h)\right) \tag{29}$$

$$= \begin{cases} [(p - w_j) \;\; h_j - \gamma_j] & \left[l_j^s(v(w,h)) + l_j^{fa}(v(w,h)) + l_j^{fo}(v(w,h))\right] \quad \forall z \leq z^* \\ [(p - w_j) \;\; h_j - \gamma_j] & \left[l_j^s(v(w,h)) + l_j^{fa}(v(w,h))\right] \quad \forall z > z^* \end{cases} \tag{30}$$

First consider the most prohibitive fixed cost of a firm that is still able to do business: $\gamma^{max} = (p - b) \;\; h^{max}$ where b is workers' home productivity (denoted b in the paper following standard notation) and h^{max} is the maximum number of weekly working hours feasible. Firms with this level of fixed costs cannot increase wages beyond $w = b$, else they make losses. Similarly, they are forced to set the number of weekly hours at h^{max}. This will strongly (negatively) affect firm size, naturally.

As fixed costs decrease, the strategy space opens, and firms can choose to use their leeway to increase wages or decrease hours to gain in size. The Burdett-Mortensen trade-off arises: On the one hand, higher wages make offers more attractive and attract more workers, increasing turnover. On the other hand, they reduce firms' margins. Now this logic applies to both increasing wages and reducing hours - the negative effect on margins arising as long as fixed costs are positive.

The minijob tax exemption then adds an additional incentive: Combinations of wages and hours that generate earnings below the threshold are potentially attractive to more workers. Firms with high fixed costs may not have this option, however. This is a route to endogenize hours choice within the current framework. In order to keep our exposition in the paper concise, we highlight the role that heterogeneity in fixed costs may play in giving rise to the different hours requirements without the formal exposition above. Note that in this model of fixed costs the *most competitive* firms (those with lowest fixed costs) offer minijobs.

J.1.3 Variation in hourly productivity as a function of weekly hours

An alternative to the above model would assert differences in (hourly) productivity. Highly routinized tasks may for example be more easily split across different individuals, making productivity more constant as (weekly) hours vary. An appropriately-chosen underlying distribution of productivity and hours should be able to generate any pattern of firms' hours choices. If there is variation across firms in the relationship between hourly productivity and weekly working hours, then we can justify different firms being active in

different markets. Without knowing the joint hours-productivity distribution it is not so easy to tell whether it is the more profitable or the less profitable firms that offer mini-jobs. Without additional structure it is also unclear what the counterfactual productivity of firms currently offering minijobs could be if the tax exemption is abolished.

J.1.4 Evidence of firm-side hours restrictions

In the models above, fixed costs of employing a worker imply firm-side preferences for specific levels of working hours. In our model, firms can express these by posting offers with specific hours. Additionally, *workers* also have preferences over working hours. We now present some empirical evidence supporting the relative importance of firm-side factors in determining working hours: Full-time workers report an unmet demand to reduce weekly working hours. This suggests full-time workers are being encouraged to work longer hours. (This is consistent with the literature's consistent finding of a full-time wage premium. For Germany, see Biewen et al. (2018).)

Table 9: Desired and reported actual hours of employees earning 800+ €

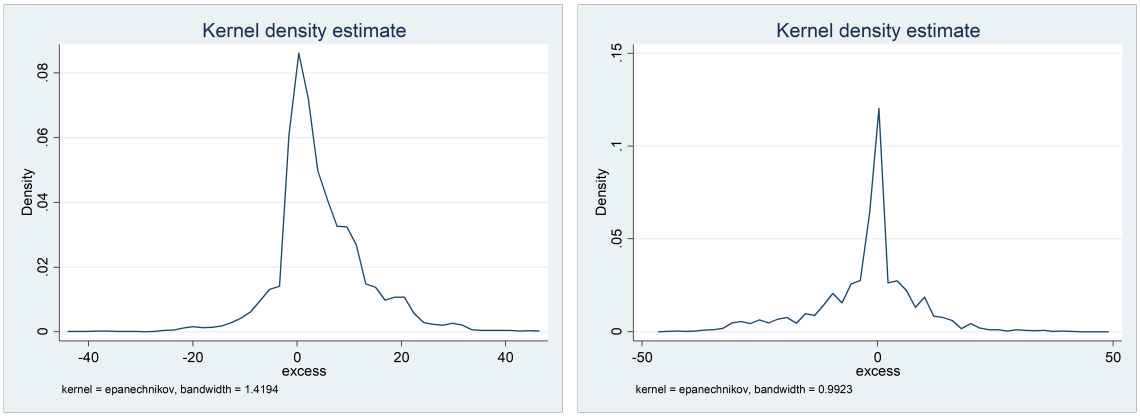
	Share of total	persons
want to work $5h+$ hours more	4.46	386
about right $(+/-5h)$	55.98	4,850
want to work $5h+$ hours less	39.57	3,428

Notes: Table reports difference between "desired hours" and "reported actual" hours of work. N=8,664 (persons) who held a regular employment; *Data source*: SOEP waves 2000-2003

We use SOEP data. The precise question is: "If you could choose your own working hours, taking into account that your income would change according to the number of hours: How many hours would you want to work?" Table (9) categorizes the difference between desired and actual working hours. Of the 8,664 individuals interviewed in SOEP (we only use one observation per person) who have a job paying more than 800€ over our observation period, only roughly 5-10% report wanting to work more than 5 hours longer, while 30-40% report that they would prefer to work 5+ hours less. The left panel of figure (4) plots the difference between actual and desired hours, with positive hours indicating greater actual than desired hours for workers with full-time jobs. On average, these workers report wishing to work 4.8 hours less than they actually do (the median is

at 3 hours). The right panel of figure (4) shows the equivalent plot for minijobber, where we see that on average minijobber would like to work 1.8 hours more per week. While interpreting these data is tricky, it seems to us to be consistent with systematic constraints on the choice of hours exist from the firm-side: Apparently, firms post job offers with more hours than workers would like. We argue that this is consistent with a competitive labor market only if there are firm-side restrictions (e.g. in terms of financing fixed costs of employment or lower productivity). Otherwise, it would be profit-maximizing for a firm to post job offers with a slightly lower wage and fewer working hours.

Figure 4: Distribution of excess of actual over desired hours for regular employees (left) and minijobber (right)



Notes: Table reports difference between "desired hours" and "reported actual" hours of work. *Data source*: SOEP waves 2000-2003

J.1.5 Implications for counterfactual policies

As noted, profit-maximizing firms will require some driver from either costs or productivity to counter the incentive to offer only minijobs. We have proposed to think about two determinants: fixed costs and productivity differentials. What would these models predict when we propose abolishing minijobs? We have already noted that the models offer starkly differing predictions as to the relative competitiveness of firms that offer minijobs:

- In the homogeneous fixed cost case, firms make equal profits. Then, the removal of minijobs would be expected to lead to a re-ordering of the offer distributions and an exit of a certain mass of firms. We offer no proof, but our intuition is based on the assumption of competition across firms in combination with free entry and exit of firms. Assume the economy was in a steady state prior to abolishing the minijob tax exemption. Since removing the minijob tax exemption implies higher taxes, some of the surplus is removed from the labor market. The firm-size and wage distributions

will adapt - this will presumably be accompanied by the exit of some firms.

- As argued above, the model with variation in fixed costs implies a threshold level of fixed costs above which firms will offer only full-time jobs. Our intuition here would be that if minijobs are abolished, the labor demand satisfied by minijobs will be transferred to other jobs (because these are the most profitable firms). However, we do not know whether they will be transformed into full-time or other small jobs.

- In the third model with productivity differentials, the predictions are less clear. Job losses may be larger or smaller compared to the models with fixed costs. Minijobs may or may not be jobs that are productive enough to cover labor costs without the tax exemption.

How do these considerations compare to the counterfactual policy simulations we report? We cover all possibilities, since we present two extreme scenarios regarding labor demand in our paper: (i) the baseline scenario we present implies the removal of labor demand from the market of small jobs when the minijob tax exemption is removed; (ii) section F considers the opposite extreme in which all labor demand remains in the market for small jobs. Our analysis allows the reader to consider a wide range of potential outcomes.

In conclusion, in the market for small jobs, including a margin of substitution of small jobs (including minijobs) to full-time jobs may influence how many jobs are removed from the market for small jobs. This makes our baseline scenario that *all* labor demand is removed from the market *more* likely. Thus if we limit our ambition to analyzing the market for small jobs, we provide results that are consistent with the possibility of firms transforming jobs into full-time contracts. We are transparent about the different paths that labor demand could take and include two polar opposite cases.

J.2 Some firms set wages in the dominated range - might this be due to short-run adjustments that are ignored?

It is correct that our model predicts no mass in the strictly dominated range. The earnings distribution we present (see figure (3) in our paper) shows that there is no range in which there is no mass. Does the empirical evidence therefore contradict our model and is this evidence of additional frictions?

We make four points. First, section J.2.1 shows that there are several dominated ranges and what this implies. Second, section J.2.2 stresses the frictions already incorporated in our model. Third, in section J.2.3 we present a different explanation for some of the earnings mass above the notch that our model does not predict. Finally, fourth, section J.2.4 tests to what extent the earnings mass that appears to be in the dominated range in our data may indeed relate to short-run optimization issues.

J.2.1 The dominated ranges

The strictly dominated range is the range of earnings in which *net* earnings fall below earnings at the minijob earnings threshold. From the point of view of individual first-jobbers who face labor taxes only above 325€, there is a very clear dominated range. The minijob earnings threshold of 325€ implies different dominated ranges for our three income tax groups: The dominated range is 326€ - 553€ for the lowest income tax group and 326€ - 830€ in the highest income tax range.[4] So we have several dominated ranges. Additionally, we also have a set of workers - the second jobbers - that has no dominated set.

To consider whether we have "too little" earnings mass in our simulation results, we thus need to adopt a firm perspective. From the point of view of the firm, offers within this range may be rational given the existence of second-jobbers. The precise range of earnings offers that are dominated from a firm perspective will depend notably on the utility distribution of offers, the relative importance of earnings compared to working hours and of course the relative size of the second-jobber group compared to the group of first-jobbers. See also the income tax schedules in the paper (figures (1) and (2)) that give the budget sets of first and second jobbers and the variance of income taxes in these groups.

J.2.2 Frictions in adjustment & wage-setting

Compared to the benchmark of unconstrained labor supply choices, we have very strong adjustment frictions: Individuals receive offers they cannot renegotiate.

[4]To make sense of these calculations, note that the rate of SSC is uniform across employees subject to SSC but varies slightly over our sample period between 42.1% and 41.28%. Given our estimated income tax brackets for type-f workers, total labor taxes thus vary between 41.28% and 61.57%, with an average of around 46%. This implies that the net income of an individual earning gross income of 326€ will lie between 191€ and 125€ and the dominated range runs from 326€ up to 553€ for those with no income tax liability up to 830€ for those in the highest income tax bracket.

Our model *can* explain why people accept wage offers in their own dominated range. An unemployed worker who finds a job in their dominated range will nevertheless be better off and accept this job.

Our model *can also* explain why firms set wage offers that will be in *some* workers' dominated range: If there are enough workers of other types in the market, this can be optimal. Thus the presence of second jobbers in a market with frictions market can explain some of the first jobbers accepting jobs in their dominated range.

Thus on the one hand, the "incentives emanating from the group of first jobbers are so large that no job offers are made with earnings just above 325€ "(p.17). On the other hand, "the presence of second-jobber in the market creates incentives for firms to make offers above that level, but within the dominated range of first-jobbers. We predict that certain firms will post earnings of around 400-450 € for example, with increasing earnings density above that."(p.17)

There is in fact a distribution of dominated ranges and individuals may rationally accept earnings offers in their dominated range. There remains some mass that the simulation cannot explain. We turn to this now.

J.2.3 Exceptions to the tax exemption

There exists an exception to the rule that to qualify as a minijob, earnings must not exceed the 325€ threshold. Temporarily exceeding the threshold for up to three months is allowed if (and only if) this occurs due to "unforeseen" circumstances. The relevant authorities give the example of a worker standing in for a co-worker who is sick. In this case, we may observe earnings above the threshold for a job that nevertheless benefits from tax exempt status.

Fortunately, we can test for these exceptions by cross-checking whether employment spells with earnings above the threshold are classified as having "tax exempt status" in SIAB. This characteristic is collected separately from the level of earnings. We find that a significant number of the spells in the earnings range that our model fails to predict are actually tax-exempt minijobs.

Table 10: Minijob spells of first-jobbers with above-threshold earnings

	tax liable (%)	tax liable (N)	tax exempt (%)	tax exempt (N)
325-349 €	22.81	2,383	77.19	8,073
350-449 €	70.25	9,190	29.75	3,891
450-550 €	92.56	11,248	7.44	904

This is surprising given that our earnings thresholds generally identify tax exempt status very well with around 95%-99% of spells classified the same way using the earnings threshold or the tax exemption status. However, apparently, the range above the notch is very specific. As we note in the paper, we "find that in the immediate vicinity (325€ - 350 €) three quarters of spells are actually declared to be tax-exempt despite reporting above-threshold earnings." (p.29) The share of these minijob spells with above-threshold earnings falls rapidly to 30% for the range 350€ - 450€ and below 10% above that. This can account for some of the mass here that our model fails to predict.

While our model cannot correctly predict the observed earnings, we feel that this is less problematic if the earnings observations in the dominated range are there for idiosyncratic reasons. We now turn to those spells where first-jobbers report earnings in the dominated range that are classified as subject to social security contributions, i.e. which "exceptional circumstances" cannot explain.

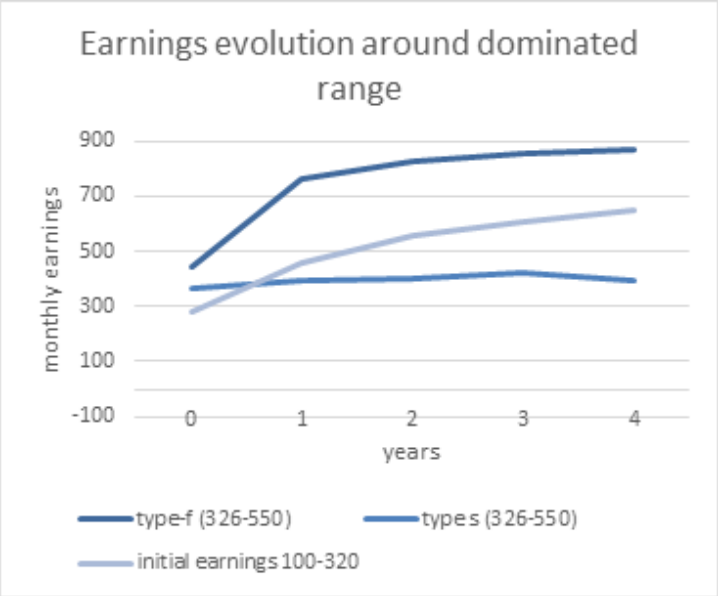
J.2.4 Earnings in the dominated region may be temporary

We consider the persistence of reported earnings by first jobbers in the dominated range. We consider the hypothesis that individuals may remain in the dominated range for only a limited amount of time. Since earnings may dynamically evolve for many reasons, we would like to have a comparison group to see whether reported earnings in the dominated range behave significantly different. Figure (5) contrast the earnings persistence of workers with earnings in the dominated range (which we set to be in the range 326 - 550€) with two other groups of individuals:

1. Second-jobbers in the same earnings range. These individuals have no incentive to leave the dominated range.

2. First-jobbers in minijobs earning 100-320 € . Since the dominated range can extend quite far depending on individual tax incidence, identifying a group earning "just below" is more obvious than a comparison group earning above the dominated range.

Figure 5: Earnings persistence across three groups in SIAB



Using the SIAB classification of tax exemption, we make sure that the jobs of first jobbers earning 100-320 € are tax exempt, and those of the other two groups not. Figure (5) shows that earnings of both first-jobber groups move beyond the dominated range, whereas the earnings of second-jobbers show much less tendency to do so. Note, however, that measures of central tendency are somewhat difficult to interpret as the dominated range can be avoided in two directions (by remaining at 325€ or by moving beyond the upper support of the dominated range). Nevertheless, the patterns may suggest that few first-jobber remain in the dominated range for a long period of time. This evidence can be viewed as confirming that short-term optimization frictions - that we do not include in the model - are the source of what we believe to be a limited phenomenon.

J.3 If some of the incidence of the income tax and SSC is on the firm, doesn't the notch for workers translate to a notch in labor cost also?

Our paper assumes that labor costs do not vary discontinuously at the minijob threshold. We first briefly note the legal basis and then turn to the question of economic incidence.

J.3.1 Legal rules on labor costs

The rules concerning minijobs were specifically designed to prevent firms from transforming regular employment into minijobs. Most importantly, whereas in the 1960s, '70s and '80s

the minijob tax exemption (which existed in a similar form) also applied to employer SSC, i.e. exempting firms from paying contributions, this was not the case during our sample period. Furthermore, employment protection, sectoral minimum wages, rules on holiday, sick pay etc. all apply in exactly the same way. As the government notes on its website: "Mini-jobbers basically have the same labor rights as full-time employees."

- **Equal treatment principle** Mini-jobbers are not to be treated less favorably than comparable full-time employees. Should full-time employees receive special payments such as a Christmas gratification for example, then a minijobber is also entitled to a gratification proportional to his hours of work.
- **Annual holidays**
  Each employee is also entitled to paid annual holidays during a minijob. If the minijobber should work six days per week, his minimum annual holidays entitlement is four weeks or 24 workdays.
- **Continued payment of salary in case of illness**
  Mini-jobbers that are unable to work due to illness through no fault of their own, are entitled to a continued payment of their regular earnings by the employer for a maximum period of six weeks. The employer also has an obligation to paying the employee his salary should there be no work due to it being a public holiday. The continued payment of the remuneration on a public holiday cannot be avoided by the employee having to work in advance of or after a day on which the employee would not normally work. Expectant mothers are also entitled to payment of a maternity leave wage and possibly an allowance in addition to the maternity benefit for the duration of the period by the employer.
- **Protection against unfair dismissal and periods of notice**
  Mini-jobbers have the same protection against unfair dismissal as full-time employees. The German Unfair Dismissal Act also applies to them.

Source: `https://www.minijob-zentrale.de/SharedDocs/Downloads/DE/Fremdsprachen/gewerblich/03_19906_informationen_fuer_an_englisch.pdf`

We think the legal position is fairly clear. Whereas abuses have been reported (withholding of holidays in particular), the extent of the phenomenon is hard to determine so that it is unclear how prominently these possibilities feature in firms' choices.

J.3.2 Economic incidence

Incidence is an interesting question in an equilibrium setting. We can think of incidence on an individual or on a more aggregate macro level. We note three points in this context: First, incidence *on an individual level* is on workers as a result of the wage posting assumption. Second, this does not imply that there is no incidence on firms, or that firms are

not affected by the tax exemption, but this occurs via the endogenous wage distribution. Third, we note that the functional distribution across workers and firms is governed by competition across firms, an empirical question in our setting.

J.3.3 Individual incidence

First, on an individual level, it is indeed the case that in our model incidence is *first and directly* on the worker: Conditional on receiving a job offer, workers subject to higher income tax will receive lower net earnings. Gross wages are fixed. The key restriction in this context concerns the modeling choice that firms and workers do not engage in bargaining but rather, firms post wages. If bargaining were possible, the net utility of individuals subject to the tax and those not subject to the tax would change. (Determining the scale of this phenomenon, i.e. how the additional surplus of a tax-free match is distributed, would require us to take a stance on workers' bargaining power.)

We believe there are strong arguments in favor of this assumption in our market. (i) The large mass of earnings for second jobbers is an argument against bargaining: If firms and workers adapted to individual circumstances, we should see no such bunching. (ii) Bargaining may be a reasonable assumption in many parts of the labor market, but for small jobs it seems least appropriate (see e.g. Cahuc et al. (2006)). Assuming zero bargaining power is also the route taken by other models of the sector of low-paid jobs (Shephard, 2017). (iii) Note that for minijobs, it is very frequent to observe job adverts specifically referring to openings as minijobs. (iv) Tax obligations vary widely across individuals - much more so than for full-time employment. Basing pay on individual tax levels would raise practical difficulties (firms would need to monitor other household income), and also fairness issues.

J.3.4 Aggregate incidence

Incidence on a more aggregate level is not only on workers. Our model predicts that the gross wage distribution depends importantly on the tax system. Because the attractiveness of (gross) wage offers depends on taxes, firms have an incentive to post wages that will not result in too high tax obligations for workers. This affects two kinds of distribution: Across firms and across worker types.

For firms, let us contrast briefly this set-up to a model in which we present all the

incidence as being on firms, with firms posting net wages. Though we offer no proof here, it is not clear to us that the outcome would be very different: Firms are still competing for workers and benefit from being able to attract many workers at relatively high margins below the tax exemption compared to above. The key determinant of the distribution of aggregate surplus across firms and workers is the degree of competition on the labor market. In our model, job-to-job transitions ensure this competition: If workers move to better jobs faster, this reduces expected profits of lower-ranked offers and entices firms to post more attractive offers.

For workers, the "no bargaining" restriction may be thought to benefit some workers (those benefiting from the tax exemption) at the cost of others. However, here the same principle applies: The wage distribution is not fixed. Our discussion of the effect of abolishing the minijob tax exemption for not directly affected workers illustrates this mechanism: "*While the population of exclusive minijobbers creates a particular incentive for firms to offer minijobs rather than higher-paying jobs, the increased competition in the market also creates an incentive for firms to offer higher wages up to the minijob threshold. In particular, workers with second jobs benefit more from the positive externality of receiving more offers than they lose due to many offers being tailored to people who benefit from the minijob threshold.*" (p.33)

In this sense, the aggregate incidence of the tax is probably best thought of as a counterfactual: What would earnings, wages and profits be in absence of the tax? This is the precisely the counterfactual policy simulation we present in the paper.

In conclusion, we would say that whilst direct nominal incidence is on the worker, our equilibrium framework implies that an important part is also on firms. Profit-maximizing firms and job-to-job transitions by workers ensure that there is competition from both sides of the market for firms to post wages that maximize joint surplus. Different models have different starting points regarding the direct incidence - e.g. it can be explicitly split in a bargaining context. It is hard to tell what effects a different set-up (e.g. with bargaining) would imply in equilibrium without a specific model in mind.

J.4 Measurement error is not allowed to push earnings beyond the minijob earnings threshold. Why?

We here briefly outline the reasons for our strategy, provide an intuitive way of understanding it and note that our strategy appears in line with the data we use. We think that the point mass at the earnings threshold is evidence that reported earnings at this point in the sample correspond precisely to the offer firms made.

Allowing measurement error in earnings to change the classification of jobs as tax-exempt minijobs or non-tax exempt jobs slightly above the threshold would be problematic. It would mean that in our simulations, firms run a risk of making very unattractive offers if their job offer ends up in the dominated range. Our treatment is thus informed by this high cost of making very unattractive offers if measurement error drives earnings beyond the threshold. Rational inattention may allow firms to set earnings levels a little less precisely for jobs paying in the earnings ranges 100 - 200 € or 500 - 600 € , but there is a strong argument for firms to be more attentive around the minijob earning threshold: They will see very few applicants if their offer is in many workers' dominated range. Rational inattention can probably be rationalized as an evolutionary equilibrium outcome: The game theoretic arguments which imply a continuous set of offered utility-levels imply that placing an offer slightly above or below the targeted level is not very problematic on the support of the equilibrium earnings distribution. By contrast, firms that post offers in the dominated range may not survive very long.

The data provider (IAB) notes that the quality of our administrative data is particularly high for characteristics that are relevant for calculating benefits. Thus earnings data - and specifically, earnings levels at the threshold - can be expected to be of higher quality for example than data on educational attainment, industry sector classification or occupation. The latter are irrelevant for calculating unemployment benefits and state pension levels and hence quality checks are less stringent.

Here a simple rationalization of our modeling strategy: Firms first decide whether to post earnings at the minijob threshold or set earnings more flexibly, as a function of productivity (using a piece rate) or actual hours worked. If they choose to set earnings at the minijob threshold, we observe earnings with certainty - otherwise the observation is subject to some error.

Finally, note that our treatment can be compared to specifying functional forms in measurement errors to avoid negative earnings, hours or wages. In fact, given the size of the utility cost for moving beyond the minijob earnings threshold, accepting a job offer with earnings slightly above the minijob earnings threshold may be associated with negative earnings for some individuals.

J.5 Firms' hours requirements are exogenously determined. So doesn't the distribution of hours depend on policy?

The assumption that firms have fixed hours requirements may appear stronger than it is. In particular, the realized hours distribution *is* endogenous to model and policy parameters. A firm with a given level of hours requirements may be more or less attractive to workers depending on their preferences, the tax schedule or the existence of tax exemptions. As a result, the number of employees working in firms with a given hours requirement will vary and the aggregate hours distribution will be affected - even if hours requirements for any given firm are fixed.

Also, as noted in section J.1, it is possible to interpret the different exogenous hours requirements as arising from different fixed costs of employment, an explanation which may be more palatable than simply assuming firms require a fixed number of working hours.

K Modeling worker behavior in the small jobs market

K.1 What if we allow workers to choose full-time and small jobs jointly?

We understand the desire to have a more complete rationalization of individuals' labor market outcomes. We make three points: Section K.1.1 considers the potential effects on our results if firms' full-time job offers change as a result of the labor market policies we consider. Section K.1.2 discusses whether holding or not holding a small job may influence either workers' rate of finding full-time employment or their likelihood of remaining in full-time employment. Section K.1.3 assesses whether we can bring some data to bear on this issue.

K.1.1 Workers' reaction to firms offering full-time and small jobs

Following on from the previous section, let us first briefly consider the effect of letting workers move to the market for full-time employment if firms change their offers as a result of policy changes. We believe that our conclusions regarding redistribution across our worker types are likely to hold *a fortiori* in this case: The only group of workers that has the option of moving to full-time employment are type-fa workers. We argue that these benefit most from a removal of the minijob tax exemption. If, additionally to having better above-threshold small job options they also have more full-time options, by revealed preference they should be (weakly) better off if firms create more of these. The only caveat is that changes in the competitiveness following a withdrawal of both workers and firms will determine the wage distribution for the remaining workers in this market (both components of κ will change). However, as we argue in the following section, we do not think this margin is very relevant for most participants in the small jobs market.

K.1.2 Allowing small jobs to influence full-time job status

The relationship in our model between full-time and small jobs is asymmetric: On the one hand, we allow workers who have a full-time job to have a different job-finding probability and a different probability of job loss compared to workers who do not have a full-time job. In this sense, we allow for a dependence of the small-job on the full-time market. On the other hand, we do not allow for small jobs to influence the likelihood of holding a full-time job.

Matches of full-time jobs are realized independently of matches in the small jobs market. We do not model the processes involved for full-time jobs. The economic rationale for this is simply that for most workers, full-time employment is by far their most important source of income, social security, health insurance and pension. Most people do not have the opportunity of *not* seeking such a job. The reasons for not taking up a full-time job are likely to be unrelated to the question of holding or not holding a small job. Reasons include student or retirement status, being married to a spouse with enough income to support two persons etc. The reasons that allow us to ignore the full-time market and focus on the small jobs market resemble the arguments made by other researchers when they focus only on the full-time market and ignore the small jobs market.

What might change if we allowed small jobs to influence whether or not workers seek

full-time jobs? Relaxing the restriction would allow workers who hold a small job to be less likely to hold a full-time job. This could come about either by job-seekers searching less intensely for a full-time job if they hold a small job or by full-time job-holders being more likely to lose their full-time job. We think that both of these are unlikely, for reasons we now present.

Do small jobs stop people finding full-time work?

Individuals seeking full-time employment are among the groups with the lowest incentives to accept small jobs. That is, if they are registered as unemployed, and have either contributed to the unemployment insurance system in the past or pass a means test. Individuals in this situation are subject to a different earnings threshold: Earnings above the level of 165€ (for individuals receiving unemployment benefits) or 100€ (for individuals receiving social assistance) lead to benefits being withdrawn one-for-one, i.e. they have a marginal tax rate of 100% above this level.

Our pragmatic solution is to exclude individuals receiving benefits: A different threshold in the tax-and-transfer system would need to be modeled for this group. In practice, and consistent with the incentives exposed, the group of unemployment benefit recipients is fairly small in our sample (around 6%).

Do small jobs encourage full-timers to quit?

With a standard utility-function in mind, we may think that a small job may reduce the incentive to seek to remain in a full-time job. While the marginal utility of leisure may indeed rise, note that our cut-off for the small job market was a level of earnings equivalent to the subsistence level - it is difficult to survive only on these earnings. Thus the key determinant of being able to *not* hold full-time employment is likely to relate to other factors and not the small job. One of the factors here will be the social security system.[5]

[5]Recall that we argue that social security contributions can be treated like taxes in the market for small jobs. (1) Individuals typically will have health-care coverage: Second-jobbers via their first job, individuals as social welfare, spouses and children can be covered by main earners in the family, retirees and students are in special regimes. (2) State pension contributions can be viewed as a tax if individuals contribute below a level that is guaranteed by the state anyway. Note that most individuals in the small jobs market are not observed in full-time employment over a long period of time - it is thus unlikely that pension contributions via small jobs will benefit these workers in retirement. (3) The replacement rate (between 60-67% of earnings) combined with the level of earnings in the market for small jobs ensures that

- Health insurance: For individuals with a full-time job who do not qualify for means-tested benefits, health insurance is provided only during the period of receipt of unemployment insurance. Giving up full-time employment with only a small job at hand will then be costly. For individuals who qualify for unemployment benefits, small jobs are also unattractive.

- Unemployment benefits when workers lose their job. The real value of these actually decreases for individuals holding a small job on top of a full-time job, thus potentially *decreasing* the likelihood that small-job holders would quit their full-time job.

K.1.3 Movements between full-time and small jobs in SIAB

Our assumptions relating to the interactions between full-time and small jobs do not preclude small job-holders taking up or losing full-time employment. Individuals that we classify as first-jobbers (i.e. currently not holding a full-time job) are not excluded from becoming second-jobbers. For two reasons it is nevertheless reassuring that relatively few first-jobbers are found to start full-time jobs:

- First, we may be less prone to believe our assumptions of limited, one-directional interactions between the small jobs and full-time jobs markets if a large number of participants in the market for small jobs also (frequently) takes on full-time jobs.

- Second, we focus strongly on the different types (f and s) when making distributional analyses. If many individuals often change their type over the lifecourse, it would be less obvious to interpret distributional findings. (Of course, many applied analyses report effects on "the unemployed" versus "the employed" or "families with young children" etc. even in absence of a lifecycle analysis).

First-jobbers accepting full-time jobs

We have analysed switches between types in the data in two ways: First, within the observation period of our estimation sample. Second, making use of all the data we have available in the SIAB.

In our sample period, we find 12,443 transitions between type-f and type-s out of a total of 422,970 spells. These transitions affect 8,040 workers (out of a total of 231,955

even for those individuals who do benefit from payments, these are very limited, especially compared to social assistance that is available for workers who are not entitled to unemployment benefits. The level of assistance varies - depending in particular on housing needs and other income sources - but can be expected to be at a similar level to the unemployment benefits of a small job.

individuals in the sample). At 2.9 transitions for 100 spells, or 3.4% of workers, the scale of switching across types appears modest.

In an attempt to obtain the best possible picture of this phenomenon we enlarge the perspective and investigate the stability of our type classification by considering the period beyond the estimation sample. We use all the data we have access to, covering 35 years, between 1975 and 2010. Recall that we do not use this larger sample to estimate our model because institutions changed considerably and data on tax-exempt jobs were not available before 1999. Note that this data limitation impede us from investigating switches from second-jobbers to first-jobbers: Participation in the small jobs market is not observable in the pre-1999 period. However, we think using this data is appropriate to answer the following question: Do individuals we identify as first-jobbers in our sample observation period accept full-time jobs before or after our observation period?

So we take the sample of workers we classify as type-f based on their behavior *in* our sample and consider their behavior *outside* our sample observation period. We find that out of the 206,499 individuals classified as type-f at the beginning of our sample period for which we have data outside of the sample period, around 18.80% or 39,392 individuals are ever observed to have a full-time job. Thus over four fifth of the individuals we label as "first jobbers" are never observed in a full-time job between 1975 and 2010.

We think these empirical considerations lend support to the proposed segmentation we operate. They are also consistent with the findings of other researchers that changes in the number of minijobbers largely operate on the participation margin, i.e. with new participants entering this market, rather than via substitution of full-time employment. Thus Carrillo-Tudela et al. (2019) note that "*non-participation outflows entirely explain the rise of marginal and part-time employment*" in Germany.

K.2 Might workers have career incentives to accept small jobs in the dominated range?

Dynamic career concerns are indeed not covered in the model. We argue that career concerns are likely less important in the small job market in Germany than in other labor markets. We address this point in three parts: First, section K.2.1 records how many moves from small to full-time jobs we observe in the data. Second, we consider the specific case

of young people in section K.2.2. Third, section K.2.3 notes that dynamic incentives do not provide a satisfactory explanation for the earnings mass in the dominated range. We use both of our datasets to investigate some aspects. Third, we discuss briefly to what extent it is possible to include these considerations into the model. Finally, fourth, we assess the effect that the mass in the dominated region may have on our results.

K.2.1 Moving from small job to full-time job

One particular manifestation of the "stepping stone" theory would be that small-job holders start a full-time job. We realize the evidence here is not conclusive, but present it nevertheless since we believe that it contrasts to what we would find in other countries with segmented labor markets.

First, as noted in the previous section, movements into and out of full-time employment are rare for participants in the small job market: In our sample period we find 7,327 transitions of workers who were only active in the market for small jobs and are then observed to start a full-time job. Putting this in perspective with the 342,847 spells by type-f workers that we observe in our sample reveals that this form of transition is not very frequent.

Second, using not only our estimation sample, but the full extent of the SIAB at our disposal, section K.1.3 finds that, of all individuals who in our sample are classified as first-jobbers, only 18.8% (39,392 individuals) are ever observed to hold a full-time job. Furthermore, not all of these transitions were direct transitions between small jobs and full-time jobs - some of the transitions may have occurred after periods of non-participation, thus making career concerns less likely.

Finally, we checked whether these transitions occurred within the same firm and find that this is the case for around one third (13,216) of the transitions. So many of these transitions occur to other firms. Of course, small jobs may also be stepping stones to jobs in other firms. It seems worth noting though, that small jobs do not appear to be a tool by which firms "test" workers. This obviously contrasts to other forms of dual labor markets (e.g. in Southern Europe).

K.2.2 Dynamic incentives for young people

While in other countries it may be the case that young people gradually enter the labor force and increase their earnings, this is not the case for German students. Indeed, health insurance coverage and benefits such as free local public transport awarded to full-time students all depend on students not working too much. The implicit notch for students is large.

Table (11) reports the results of a simple multinomial logit of minijobbers' transition propensities. Among 6,129 individuals interviewed about their transitions in our sample observation period, we find that being a student strongly negatively predicts transitioning out of a minijob into regular employment and positively predicts moving to non-participation.[6]

The situation is different for young people who are not studying: Apprentices clearly do face dynamic incentives, and earnings for apprentices vary widely across different professions. Earnings levels can be very low in some tracks, even where later earnings may be substantial. Fortunately, apprentices are clearly labeled in the SIAB data, and we exclude them. They are not in competition on the labor market for small jobs, even where their earnings may lie within this range. They have a clear career progression path and it would be very unusual to hold a small job on top of the dual activities of working and further education that apprentices follow.

K.2.3 Dynamic incentives and the dominated range

Our model does not exclude that some workers accept low-paid employment which is dominated by earnings at the threshold. (The model does not require dynamic incentives to allow for this.). And individuals may have very strong incentives to take on a job. Firms may have market power and may extract rents as a result of firm-specific human capital or learning. However, it seems to us that none of this can explain what appears to be a coordination failure between the two agents - firms and workers - who are leaving money on the table if their match pays earnings in the dominated range.

[6]Note that since our data extends eight years beyond the end of the observation period we use in our sample (results reported above based on data until 2010), some young people in our sample may not yet have terminated their educational careers by this point in time. Their progression to full-time employment may then not be recorded in this data. Note also that we have not set up the SOEP in a spell format for this exercise - these are thus transition probabilities between yearly surveys.

Table 11: Transition probabilities out of minijobs (multinomial logit)

	coefficient	standard error	z-value
Transition to Tax-liable regular employment			
student	-1.088361	0.1561332	-6.97
high-earning spouse	-0.8287842	0.1186645	-6.98
retiree	-1.563727	0.2891025	-5.41
Transition to non-participation			
student	0.748515	0.1132525	6.61
high-earning spouse	-0.0320685	0.1118891	-0.29
retiree	0.7104824	0.1496767	4.75

Notes: N=6,129; Multinomial logit with base category: minijob employment. Std. Err. clustered by individuals. Using (cross-sectional) weights provided by SOEP. Additional controls: age, age squared, female, years of education, log hourly wage, East Germany. *Data source*: SOEP waves 2000-2003 (based on yearly surveys).

K.3 A fraction θ of type-f workers never accept jobs with earnings exceeding the threshold (type-fo). Couldn't this heterogeneity be better captured via leisure preferences (α)?

How would heterogeneity in α act? Individuals who have other important commitments - as is the case for many in the market for small jobs - would be modeled as having a greater preference for leisure. However, as long as these workers also prefer higher wages, we would find a counterfactually strong incentive for firms to post higher wages to attract more workers. (This occurs despite the standard trade-off relating to profit margins per worker, in line with the well-known prediction of increasing earnings density of earnings in the Burdett-Mortensen model.) Since this is contrary to what we observe, what could pose limits on this incentive?

K.3.1 Preferences for leisure, consumption... and earnings?

If we wanted to interpret the data in terms of preferences, we would note that in our market, workers' utility surely depends on hours and consumption. Unusually, we would need to recognize that (some) workers have strong preferences *not to earn too much*. This is because these groups may forfeit their pensions or the possibility to study if they earn too much. This implies that we would require a second parameter beyond α, which would be

a preference over earnings alongside our consumption-leisure parameter. While it would surely be possible to specify preferences that rationalize the wage offer distribution, it seems to us that for any *standard* preferences to justify the large point mass at the threshold, the tax schedule and hours requirements are important: Only these can prevent firms from offering earnings beyond the threshold that would be attractive to workers with a strong preference for leisure.

As an example, we might have the following two persons with the same preferences regarding the leisure-consumption trade-off *below* the minijob income threshold: Person (A) prefers not to work at all if income exceeds the minijob income threshold. Person (B) would happily exceed the minijob earnings level (subject to the usual lack of attractiveness of earnings immediately above the minijob earnings threshold). Their job-to-job transitions and hours-leisure choices below the threshold will be the same, and yet we have important heterogeneity.

K.3.2 Discriminating between preferences & constraints

So we view the heterogeneity not so much on the side of preferences with respect to leisure, but rather as differences in budget constraints - with material constraints that not only emanate from the tax schedule (see section K.4).

While we present no test here, what empirical evidence could help discriminate between the theories of heterogeneity in preferences over leisure versus heterogeneity in implicit taxation above the minijob threshold?

Heterogeneity in α would manifest itself in variation in job-to-job transitions throughout the earnings distribution. Individuals who value leisure highly would be less willing to move to higher earnings if this implied increased hours, whereas other individuals would be able to do so. While in this setting we would still expect a certain reluctance of individuals to move beyond the threshold, we would expect all individuals to *eventually* move beyond the threshold at some point in their labor market careers. The non-linearities of the budget constraints should show up in transition probabilities at different points in the wage-hours distribution, but differentiating between the two may be difficult in practice.

Instead of testing these hypotheses, we limit ourselves to some suggestive evidence on the (relative) reluctance of certain groups to move from minijobs to higher-paid employ-

ment. Using data from the SOEP we find, first, that very roughly one half of the population of minijobbers is either retired, university students and married to rich spouses. Retirees and students make up over 30% of type-f workers, see table (12). We expect these to face serious costs of moving beyond the threshold even if this is not visible in their tax schedule. In line with this, table (11) also shows that individuals from these groups are significantly less likely than others to move from minijobs to jobs with greater earnings, and significantly more likely to move from minijobs out of the labor market.

On balance, we feel that while the model parameter theta is rather special, we do not see an adequate way of modeling this (maybe rather special) market in a more parsimonious or standard route using preferences.

K.4 Do we require heterogeneity in type-f workers (θ)? Even without heterogeneity your model should generate a spike in the earnings distribution given the discontinuity & the budget constraint.

Our model would indeed produces a spike in the earnings distribution even without θ - if enough individuals have large notches in their tax schedules. Maybe if we observed tax liability better, the correct tax schedules could generate strong enough incentives for enough firms to set earnings at the threshold? We have a lot of sympathy for this view. Indeed, in the data we impute zero income tax for a large share of type-f workers. Although they face a combined labor tax (SSC + income tax) of around 40%, firms still have an incentive to post high earnings to attract these workers (see figure (1) in the paper). Do individuals in this market maybe face higher taxes than we think?

Imputing taxes is not easy - we confirm the difficulties that others have found in using SOEP data to impute taxes in SIAB (see (Junge, 2017)). However, imputing zero income tax liability is maybe more robust than a precise level of income tax. In our market, SSC are the more important component and here we face no uncertainty. Rather, we think other non-tax, but material, constraints are important. The SOEP tells us that retirees and students make up around one third of the minijobber population, as shown in table (12). As noted, these demographics not only face the labor tax (income tax & SSC) notch, but much greater penalties: Retirees may jeopardize their retirement income and students may forfeit their health insurance, making it *de facto* prohibitive to earn more.

Viewed this way, our modeling strategy is informed by the empirical observation that

the observable variation in budget constraints is insufficient to account for the point mass we observe in the earnings distribution. We believe that the reason for this lies in known additional costs to specific groups in the population which arise when they increase their earnings beyond minijobs. The question to ask is: Would a large proportion of students or retirees accept to work more hours if only the wage rate were high enough? We believe that this to be unlikely. The way to make jobs more attractive for these groups is to reduce hours, but remain within the earnings limit which has other material benefits for this demographic.

Though unusual, we believe for this specific market this segmentation is the correct modeling strategy given the composition of the individuals who only hold a small job. We allow the data to inform us how large the fraction of the population is that faces prohibitively high costs of accepting offers with earnings beyond the minijob threshold. Note that our model nests the case in which the fraction of type-fo workers is zero, and the explanatory power of the budget constraint is sufficient to fit the data well.

K.5 The share of individuals who never accept jobs beyond 325€ is estimated to be equal to about 37%. This seems very large?

To assess whether this figure is credible, we compare the estimated value of 37% to the composition of the first-jobber workforce. We can also investigate the propensity of specific groups we might believe to be of type-fo to accept jobs with earnings greater than the threshold. Before we do this, we would like to recall two points:

First, individuals need not be subject to a *marginal* tax rate of 100% to make small jobs above the minijob threshold wholly unattractive. We are faced with a *notch* rather than a kink here: Consider the estimated 11% of the first-jobber population for which we calculate a marginal income tax rate of around 19.47%. Combined with the social security contributions this combines to a total labor tax rate of over 60%. Given that the first 325€ of earnings are also subject to these 60%, the dominated range for this group extends to 830€ - i.e. gross incomes below this level are dominated by minijobs with gross (= net) incomes of 325€ . See also the tax schedules, figures (1) and (2) in the paper.

Second, for certain groups the prohibitive tax will not arise directly via the tax system. As we note in the paper, e.g. if students lose their student status and retirees jeopardize their retirement income.

K.5.1 Composition of the small job workforce

We now assess whether a share of 37% is realistic with respect to the share of first jobbers that is likely to find themselves faced with either prohibitive taxes or other material disincentives. We use the SOEP sample to address this issue since this provides us with the necessary information on individuals' circumstances, both to calculate (partially household-based) taxation but also other circumstances such as education and retirement status.

We focus on individuals in our sample timeframe who held a minijob as only employment (first jobber). Table (12) shows that 7.2% of these are retirees, 25.4% are in education (we have included both school and university students in this category), while 22.4% have a spouse earning at least 2000€ monthly. Given that there is little overlap across these categories (we are basing the high-earning spouse on labor income), we find that 54% of minijobber are in any one of these three categories. It seems to us that with these orders of magnitude in mind, a figure of 37% of type-fo workers appears realistic.

More recent empirical evidence suggests that the picture has not fundamentally changed. Thus the German Statistical office published an analysis for the year 2010 with the three largest categories the following composition: Housewives (sic): 35%; Students, pupils: 20%; Retirees: 22%.

Table 12: Composition of minijobs held by first-jobbers

	Share of total sample (persons)
Student	0.2545053
High-earning spouse	0.224789
Retiree	0.0723989
any of the above	0.540038

Notes: N=4,104 (persons) who in any of waves held a minijob as first job; *Data source*: SOEP waves 2000-2003

K.5.2 Who moves beyond the threshold?

We have argued that it is not unrealistic that 37% of individuals face incentives strong enough to prevent them from accepting jobs beyond minijobs in the market for small jobs. This is a prediction about the propensity of accepting employment beyond the minijob threshold, i.e. subject to social security contributions.

Using the same SOEP sample as above, table (11) reports the results of a simple multinomial logit model. We study the determinants influencing whether minijobbers move to regular tax-liable employment or to non-participation. Consider the three demographics that we believe are well characterized by the behavior modelled as type-fo - namely students, retirees and minijobbers with high-earning partners. All three groups display significantly lower likelihood of accepting employment with earnings greater than 325€ . Students and retirees are also significantly more likely to move to non-participation than others. While the sample is small, we think these findings somewhat support the plausibility of our estimates.

K.6 Do workers accept all offers? Shouldn't workers have a reservation wage?

As noted, employed individuals only accept offers with utility levels above the utility of their current job - they operate a reservation utility level. Job-seekers without a small job accept all offers. This might appear a very strong assumption. It is however consistent with our equilibrium model and empirically arguably more acceptable in our context than in others. We take this route for the following reasons, which we discuss in section 3.2 in the paper.

K.6.1 In equilibrium, all offers find takers

How would firms react to workers operating a common reservation wage - or, in our case, reservation utility? There would be no purpose in proposing offers below this value. A *common* reservation wage (or rather, reservation utility) policy would make little sense in an equilibrium model: Offers below this level would not be offered and thus the constraint would not be binding. We would not be making a mistake by ignoring the reservation utility.

K.6.2 Job acceptance rates *do* vary

We do allow for different rates of job-finding as a result of the presence of a full-time job or not - as a result of differential search intensity rather than by postulating reservation utilities. We are not the only study to focus on search intensities rather than reservation policies - Lentz and Tranaes (2005) also argue that the results of their "analysis can be carried over directly to conclusions about reservation wages instead of search intensities".

Introducing a reservation wage/utility policies would require introducing additional heterogeneity across individuals - in the value of outside options b maybe. It is unclear to us what an empirical analogue for this may be - a typical candidate relates to unemployment benefits. We exclude the relatively small group of participants in the market for small jobs who receive benefits. Of course, there are other techniques for identifying reservation wages, but these are not trivial and "*[i]dentifying reservation wages (in a fairly limited sample window) is challenging*". (p.13).

Why more complexity?

Also, what do we wish to achieve by modeling heterogeneity in reservation wages? A frequent motivation in the literature has been to account for variation in wages. We can account for wage variation using a different feature of the labor market: "on-the-job search is (...) a way of explaining why firms offer workers jobs which generate utility beyond the reservation level - higher wages make jobs more attractive for workers in employment who may be tempted to change jobs". (p.13).

We think it is noteworthy that it is possible to fit a decent wage distribution without reverting to heterogeneity in reservation utility, which in our case would presumably be imprecisely estimated or based on ad-hoc assumptions - related to observable individual characteristics maybe?

K.7 Shouldn't all full-time workers be considered potential type-s workers?

In our sample, we do not include individuals who never hold a small job in the analysis. As with any Poisson process, it may be the case that workers are interested in a second job and search for many, many years without success. These workers would then be falsely

excluded from the sample. This fundamental problem arises in many empirical job search studies: Determining who exactly constitutes the population of job-seekers is hard, especially in absence of information on job search intensity. Apparently simple rules such as restricting the analysis to registered unemployed do not solve the issue given that this misses other jobseekers who are not registered unemployed.

One option would indeed be to assume that all workers are potential type-s workers. Would it be a better assumption though? Let us consider this question for the labor market in general. We know of no paper that includes the whole population (including students, pensioners of all ages, parents of very young children, people on disability pensions etc.) in their sample. Why? Because very many individuals do not take part in the labor market. If we assume their non-participation is a result of taste, taxes or job offer arrivals, what would happen?

As we change the definition of job-seekers, the likelihood of job offer arrivals decreases. If we stick with homogeneous preferences, the estimated job-finding rate would become very low, even if we include unobservable variation in taxes due to unobserved implicit taxes (of up to and exceeding 100%) for students and pensioners. As a result, we think that we would not model the dynamics of the labor market very well - or the competitive pressures. Such a low job-offer arrival rate would imply that firms would be able to set very low wages, since the risk of losing a worker to a higher-paying firm is very low. Recall that in the limit, firms set $w = b$ as the chance of workers moving to another firm tends to zero. We would surmise that the fit of the model would not be good. Movements in the labor market between jobs with different hours and earnings packages should inform the leisure-consumption trade-off if we want to achieve as good a model fit as possible. To see this, imagine workers only cared about wages, not hours. Then, we would see a strict ordering according to wages on job-to-job transitions (subject to measurement error) and hours would be random. This would change the incentives for firms with very low hours to post low wages (the utility function is not linear). Again, to model the dynamics on the labor market - between labor market participants and firms, it is useful to focus on the tastes and informational frictions (job offer arrival rates) relevant for the participants. Finally, of course we could add an additional taste parameter relating to participation in the labor market, but this would not appear to make the model any more elegant or simpler.

If it were simple to designate the set of job-seekers, there would be no direct flows between non-participation and employment - all workers would first enter the pool of job-seekers available for work before starting a job. However, these flows have been found to be rather large, for Germany most recently by Carrillo-Tudela et al. (2019). We realize that there is a particular challenge in this market - greater than for other markets - because participation is lower, and governed by factors that are less easily observed. Nevertheless, this is an interesting market to study and we are adopting fundamental selection principles that other labor market researchers also adopt when they exclude non-participants.

Including all workers with a full-time job as potential type-s workers would not give us a good estimate, we believe, of the average duration of job-seeking for active participants in the small job market, since it would be strongly influenced by non-participants who never take on a small job. We believe that we treat this issue head-on by offering an alternative definition of the population of job-seekers in section H, show how this affects results and refer to this uncertainty also in discussing our results. There is, unfortunately, little other evidence on empirical job offer arrival rates for the market of small jobs that we can compare our estimates to.

K.8 Workers are classified as type-f or type-s in the counterfactual policy simulations. Are you assuming these population sizes are policy invariant?

We have made assumptions about the reactions of different types of worker to policy changes, but we have not assumed that the shares of the groups are policy invariant. In a very obvious way, type-fo workers react to policy and the share of type-f workers will fall if the minijob tax exemption is abolished. We discuss in turn our model assumptions concerning the population of type-fo workers to more or less generous tax exemptions.

K.8.1 Reaction of type-fo workers to a more generous tax exemption

Our section on "Smoothing the tax schedule" considers how type-fo workers react to a policy of raising the level of the tax exemption (increasing the threshold value). We state our strategy on page (p.34): "we postulate that the group of exclusive minijobbers (type-fo workers) gradually choose not to participate in the labor market when earnings - and therefore average taxes - increase. (...) Type-fo workers will thus also accept some jobs above the threshold. We assume a uniform distribution of new earnings cut-offs beyond which different workers choose no longer to participate." (p.34)

The specific implementation may of course be criticized, but we note that the challenge we face in determining the effect on the workforce of such policies is not specific to our context. In fact, it can simply be seen as a manifestation of the basic non-recoverability of the latent distribution of matches that may emerge under policies that increase employment - the same non-recoverability that Flinn and Heckman (1982) discuss with respect to the wage distribution.

K.8.2 Reaction of type-fo workers to removing the minijob tax exemption

The concept of type-fo workers is indeed very relevant for policy. We believe that it is reasonable to include in a model of the small job market a group that will exit the labor market if the tax exemption is removed. What makes us think this?

Considering the profiles of type-fo workers (to fix ideas: students, pensioners, spouses of high earners), note that unless other regulations are relaxed, removing the minijob tax exemption will imply considerable costs for these individuals. Even a marginal tax rate below unity creates a dominated range beyond our market for small jobs. For pensioners and students, who face material penalties (see table 12 above), the implicit marginal tax is likely to be greater than unity. They may be better off not working than taking on a minijob. Indeed, we now argue that this is what occurred in the past. (And for retired individuals, this is also the more common pattern across Europe, where Germany has seen a more rapid increase in participation of old-age workers.) We now consider some evidence that participation in this market has varied greatly and in particular that entry and exit from the labor market rather than substitution of a minijob by another type of contract is the typical reaction to making these jobs more or less attractive. We think focusing the model on the extensive margin reaction is a reasonable assumption for the scenario of abolishing the tax exemption.

Since we estimate structural parameters using an equilibrium model, we have focused on a period with no changes in policy. To consider the reactions of minijob holders to changes in the attractiveness of this type of employment we thus need to look beyond our sample period.

K.8.3 Participation response to minijob reform 2003

The reactions to the reform of 2003 appear to constitute prima facie evidence in favor of our assumptions. As we note in setting out the institutional and historical context:

In 2003, as part of the "Hartz" labor market reforms, access to minijobs for employers was simplified and the minijob earnings threshold increased. We do not analyze the post-reform period in particular because workers who took on a minijob as second job could now also benefit from the tax exemption, making our strategy for identifying equilibrium effects less clean. Furthermore, the number of minijobs grew rapidly, especially in the form of second jobs, making an equilibrium analysis less credible. The post-reform period is however interesting because it highlights the specific role of minijobs in the wider labor market. In particular, minijobs have not been identified as a major contributing factor to the fall in German unemployment throughout the early 2000s.[7] *The minijob reform occurred largely on top of changes in the composition of the existing labor force, in other words, "non-participation outflows entirely explain the rise of marginal and part-time employment" (Carrillo-Tudela et al. (2019)). This makes sense as minijob workers often belong to groups with low labor market attachment.* (p.4)

K.8.4 Response to introduction of a national minimum wage 2015

In 2015, a national minimum wage was introduced in Germany. This also changed the attractiveness of minijobs: Since many minijobs paid hourly wages below the minimum wage, the combination of minimum wage and threshold for minijob tax exemption forced firms and workers to decide whether to continue operating as a tax-exempt minijob with the same amount of earnings, to transform the minijob into a regular (non-tax-exempt) job, or to end the match. The prediction based on our model would be that type-fa workers would favor moving to higher earnings, while type-fo workers would prefer continuing to work in minijobs and would then work fewer hours. The SOEP sample allows us to assess this prediction by looking at the transitions out of minijobs after the introduction of the minimum wage in 2015.

Table (13) shows the determinants of transitions out of minijobs for 2,267 individuals

[7] Candidates that have been identified include collaborative industrial relations (Dustmann et al. (2014)) in the face of competitiveness issues, increased efficiency of matching unemployed workers to jobs (Launov and Wälde (2016)) and the reform of the unemployment insurance (Krause and Uhlig (2012)).

in minijobs in the years 2000-2018. We focus on the change in transition probabilities in the years following the introduction of minimum wages - 2015 and 2016:

Table 13: Determinants of transitioning out of a minijob

Multinomial logit	coefficient	standard error	z-value
Transition to tax-liable regular employment			
year 2015	-0.3714073	0.4245612	-0.87
year 2016	2.022575	0.4730129	4.28
Student, retired or high-w spouse ("type-fo")	-0.2505456	0.2666593	-0.94
"type-fo" x year 2015	-0.329794	0.7747354	-0.43
"type-fo" x year 2016	-1.49053	0.7073965	-2.11
low hourly wage	-0.7711082	0.266066	-2.90
low hourly wage x year 2015	1.805433	0.7210568	2.50
low hourly wage x year 2016	0.1173301	0.7204899	0.16
Transition to non-participation			
year 2015	-0.3716344	0.5480575	-0.68
year 2016	1.514867	0.647386	2.34
Student, retired or high-w spouse ("type-fo")	-0.1103578	0.2538399	-0.43
"type-fo" x year 2015	0.1299113	0.6417602	0.20
"type-fo" x year 2016	-1.196523	0.7683707	-1.56
low hourly wage	0.502788	0.2735991	1.84
low hourly wage x year 2015	1.085849	0.6391646	1.70
low hourly wage x year 2016	-0.1503342	0.7798866	-0.19

Notes: N=2,267 (1,663 individuals); Base category: Mini-job employment. Std. Err. clustered by individuals. Using (cross-sectional) weights provided by SOEP. Additional controls: age, age squared, female, child in household, interaction female & child in household, years of education, log hourly wage, East Germany, dummy for year 2017. *Data source*: SOEP waves 2000-2018 (based on yearly surveys).

First, we find both a significant increase in transitions from minijobs into regular employment and non-participation in 2016. In our framework we can make sense of this in terms of flows of type-fa and type-fo workers.

We then interact the year dummies for the years after the introduction of the minimum wage with two further variables: First, low hourly wages. This dummy indicates whether declared hourly wages prior to the introduction of the national minimum wage were below the level at which they were introduced. These are individuals who we expect to be more

strongly (directly) affected by the introduction of the minimum wage. We find that while in general low-wage workers are less likely to transition to full-time employment, in the year of the introduction of the minimum wage, the effect is more than reversed. This indicates that there may be an effect of minijobs being transformed into regular employment. We would interpret these as type-fa workers. However, we also find suggestive evidence (significant at 10% level) of an increase in transitions to non-participation by low-wage workers out of minijobs - both in general but also in 2015 in particular.

Finally, we consider an indicator for a group of workers we consider to be more likely to be type-fo: We create a dummy for students, pensioners and individuals with high-earning spouses. We find that this group is significantly less likely to transition to full-time employment in the year of introduction of the minimum wage, largely offsetting the positive effect across the whole of the sample.

We believe the results - despite the small sample size - support the idea that there is a group of individuals for who the participation margin is particularly important. The data also support another perspective on type-fa and type-fo workers: The former are willing to work more if they are given the chance, while the latter are not. We cannot distinguish here whether this is due to preferences or unobserved heterogeneity in marginal taxes, but a policy that made minijobs relatively less attractive seems to have been met with opposing reactions by different groups in the population of minijobbers. Our model can make sense of this.

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