Online Appendix
Equilibrium effects of making work pay policies: Evidence from Germany

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1 Discrete variation in working hours

In the main text, we assume continuous variation in hours worked. We here discuss the model equilibrium with discrete variation in hours: Weekly hours worked are in one of two categories h_k, such that $k \in (1,2)$. In the market for low-paid jobs, this may correspond to 10 and 20 hours of working. We later consider how this model generalizes to three or more hours categories.

Firms set wage rates, w, workers derive utility from consumption and leisure. In order to simplify notation, we follow Shephard (2017) and define $q_2(w) = w$ and $U(q_1(w), h_1) = U(q_2(w), h_2) = U(w, h_2)$, so $q_1(w)$ is a function that denotes the wage rate that makes individuals indifferent between working with few ($k = 1$) hours at $q_1(w)$ or working more ($k = 2$) hours at w. Depending on preferences, individuals may require a low-hours wage premium or accept a low-hours wage penalty.

1.1 Worker mobility

The flow into and out of small jobs must balance (see appendix A of the main text). The flow of workers of type $j \in (s, 0a, 0m)$ from and into jobs with hours h_k and wage rate w is

$$D^j(w) g_k^j(q_k(w)) e_k^j = \lambda^j f_k(q_k(w)) \left(u^j + G_1^j(q_1(w) - \epsilon) e_1^j + G_2^j(w - \epsilon) e_2^j \right) \tag{1}$$

with $D^j(w) = [\delta + \lambda^j((1 - F_2(w)) + (1 - F_1(q_1(w))))]$ for $j \in (s, 0a)$. Equation 2 states the corresponding definition for workers of type $0m$ who do not accept jobs with wage rates larger than w_k^*.

$$D^{0m}(w) = \begin{cases} [\delta^0 + \lambda^0((F_2(w_2^*) - F_2(w)) + (F_1(w_1^*) - F_1(q_1(w))))] & \forall w \leq w_2^* \text{ and } q_1(w) \leq w_1^* \\ [\delta^0 + \lambda^0(F_2(w_2^*) - F_2(w))] & \forall w \leq w_2^* \text{ and } q_1(w) > w_1^* \end{cases} \tag{2}$$

The LHS of equation 1 pertains to workers who leave a job in a sector k with wage $q_k(w)$. For $k = 2$ this group consists of workers who move from sector 2 to sector 1 ($\lambda^j(1 - F_1(q_1(w)))g_2^j(w)e_2^j$ for $j \in (s, 0a)$), who move to a better paying job within sector 2 ($\lambda^j(1 - F_2(w))g_2^j(w)e_2^j$ for $j \in (s, 0a)$) and who lose their small job ($\delta^j g_2^j(w)e_2^j$). The RHS pertains to workers who start a job in sector k with wage rate $q_k(w)$. For $k = 2$ this consists of workers who move from sector 1 to sector 2 ($\lambda^j f_2(w)G_1^j(q_1(w) - \epsilon)e_1^j$), who changes jobs within sector 2 ($\lambda^j f_2(w)G_2^j(w - \epsilon)e_2^j$) and who were previously job-seeking ($\lambda^j f_2(w)$). The overall flow (i.e. both sectors) due to separations from jobs with wage rate of no greater than w is:

$$\begin{aligned} &(G_1^j(q_1(w))e_1^j + G_2^j(w)e_2^j)D^j(w) = \lambda^j u^j F_1(q_1(w)) + \lambda^j u^j F_2(w) \\ &= \lambda^j u^j + \lambda^j u^j - \lambda^j u^j(1 - F_1(q_1(w))) - \lambda^j u^j(1 - F_2(w)) \end{aligned} \tag{3}$$

Recall that flows of workers entering and exiting small jobs must be equally large in equilibrium, i.e.

$$\delta^j(n^j - u^j) = \begin{cases} \lambda^j u^j & \text{for } j = s \\ \lambda^j u^j F(z^*) & \text{for } j = f \end{cases} \tag{4}$$

We then have

$$G_1^j(q_1(w))e_1^j + G_2^j(w)e_2^j = \frac{\delta^j n^j - u^j D^j(w)}{D^j(w)}. \tag{5}$$

By combining equations 1 and 5 we obtain

$$g_k^j(q_k(w))e_k^j = \frac{\lambda^j f_k(q_k(w)) \left[u^j + \frac{\delta^j n^j - u^j(D^j(w-\epsilon))}{D^j(w-\epsilon)}\right]}{D^j(w)} \tag{6}$$

1.2 Firm size

The number of workers of type j in steady-state employed at a firm in sector k which offers wage rate $q_k(w)$ is

$$\begin{aligned} l_k^j(q_k(w)) &= \frac{g_k^j(q_k(w))e_k^j}{f_k(q_k(w))} \\ &= \frac{\lambda^j \delta n^j}{D^j(w)D^j(w - \epsilon)}. \end{aligned} \tag{7}$$

The steady state firm size is then

$$l_k(q_k(w)) = l_k^s(q_k(w)) + l_k^{0a}(q_k(w)) + l_k^{0m}(q_k(w))$$
$$= \begin{cases} \frac{\lambda^s\delta^s n^s}{D^s(w)D^s(w-\epsilon)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(w)D^{0a}(w-\epsilon)} + \frac{\lambda^0\delta^0 n^{0m}}{D^{0m}(w)D^{0m}(w-\epsilon)} & \forall w \leq w^* \\ \frac{\lambda^s\delta^s n^s}{D^s(w)D^s(w-\epsilon)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(w)D^{0a}(w-\epsilon)} & \forall w > w^* \end{cases}. \quad (8)$$

Following the standard arguments of profit equalization, we find the following (the reasoning is parallel to the case without hours variation):

Proposition (A1) *There can be (at most one) mass point in the wage offer distribution at the threshold in each sector, i.e. at wages* $w_k^* \equiv \frac{z^*}{h_k}$.

Sketch of Proof: The following argument closely mirrors the argument in the case of homogeneous hours. We compare profits at the threshold value with profits above. We find that if there exist offers above, there must be a mass point at the threshold.

The profit of a sector k firm offering wage rate $q_k(w)$ can be expressed as $\pi_k(q_k(w)) = (ph_k - q_k(w)h_k)l_k(q_k(w))$. We first state the profits of a type-2-firm, assuming that $q_1(w_2^*) \leq w_1^*$.

$$\begin{aligned}
\pi_2(w_2^*) &= \frac{\lambda^s\delta^s n^s}{D^s(w_2^*)D^s(w_2^*-\epsilon)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(w_2^*)D^{0a}(w_2^*-\epsilon)} + \frac{\lambda^0\delta^0 n^{0m}}{D^{0m}(w_2^*)D^{0m}(w_2^*-\epsilon)} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s + \lambda^s((1-F_2(w_2^*)) + (1-F_1(q_1(w_2^*))))][\delta^s + \lambda^s((1-F_2(w_2^*-\epsilon)) + (1-F_1(q_1(w_2^*)-\epsilon)))]} + \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0 + \lambda^0((1-F_2(w_2^*)) + (1-F_1(q_1(w_2^*))))][\delta^0 + \lambda^0((1-F_2(w_2^*-\epsilon)) + (1-F_1(q_1(w_2^*)-\epsilon)))]} + \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0 + \lambda^0(F_1(w_1^*) - F_1(q_1(w_2^*)))][\delta^0 + \lambda^0(f_2(w_2^*) + (F_1(w_1^*) - F_1(q_1(w_2^*)-\epsilon)))]}
\end{aligned} \quad (9)$$

Evaluated marginally above the threshold, profits are

$$\begin{aligned}
\pi_2(w_2^*+\epsilon) &= \frac{\lambda^s\delta^s n^s}{D^s(w_2^*+\epsilon)D^s(w_2^*)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(w_2^*+\epsilon)D^{0a}(w_2^*)} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s + \lambda^s((1-F_2(w_2^*+\epsilon)) + (1-F_1(q_1(w_2^*+\epsilon))))][\delta^s + \lambda^s((1-F_2(w_2^*)) + (1-F_1(q_1(w_2^*))))]} + \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0 + \lambda^0((1-F_2(w_2^*+\epsilon)) + (1-F_1(q_1(w_2^*+\epsilon))))][\delta^0 + \lambda^0((1-F_2(w_2^*)) + (1-F_1(q_1(w_2^*))))]}
\end{aligned} \quad (10)$$

Equations 9 and 10 show that the equal profit condition can only hold if there is a mass point in the offer distribution of sector 2 at w_2^*. By symmetry, note that the same argument can be made with respect to a type-1 firm. However, if the utility of a threshold offer lies in the "gap area" due to a threshold in another sector, it may be the case that there is no mass point in that sector. This explains the restriction "at most one" in Proposition (IV) and completes our discussion.

We now consider the influence of thresholds in other hours sectors on the wage distribution. Consider a firm of type 2, i.e. seeking a worker to work for h_2 hours. The impact of a potential mass point in the offer distribution of sector 1 at w_1^* depends on the relation between w_2^*, $q_1(w_2^*)$ and w_1^*.

Proposition (A2) *There will be no wage offers at wage levels (and in a certain interval below this level) that offer the same utility as is available at threshold wages $w_{j\neq k}^*$ in other sectors.*

The intuition for Proposition (A2) is the following: It is a dominated strategy to offer a wage rate that is equal in utility to an offer made by several other firms. A slightly higher offer will attract all workers from these firms at only marginal cost. By Proposition (IV), wage offers at earnings thresholds generate mass points in the wage offer distributions. Thus for example a type-2 firm will offer a wage rate slightly larger than $\tilde{w}_2$ (where $U(\tilde{w}_2, h_2) = U(w_1^*, h_1)$.) in order to additionally attracts workers from this positive mass of sector 1 firms. This implies that there must be a gap in the wage offer distribution at $\tilde{w}_2$. How much below this utility value an offer can be sustained in equilibrium will depend on the parameters of the model in an analogous way to the potential existence of offers below the threshold offer in the homogeneous case.

Sketch of proof: Let $\tilde{w}_2$ denote the wage rate which satisfies $U(\tilde{w}_2, h_2) = U(w_1^*, h_1)$. If $\tilde{w}_2 > w_2^*$ the profits of a sector 2 firm offering wage rate $\tilde{w}_2$ and slightly above are:

$$
\begin{aligned}
\pi_2(\tilde{w}_2) &= \frac{\lambda^s \delta n^s}{D^s(\tilde{w}_2)D^s(\tilde{w}_2-\epsilon)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(\tilde{w}_2)D^{0a}(\tilde{w}_2-\epsilon)} + \frac{\lambda^0\delta^0 n^{0m}}{D^{0m}(\tilde{w}_2)D^{0m}(\tilde{w}_2-\epsilon)} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))][\delta^s+\lambda^s((1-F_2(\tilde{w}_2-\epsilon))+(1-F_1(w_1^*-\epsilon)))]}+ \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))][\delta^0+\lambda^0((1-F_2(\tilde{w}_2-\epsilon))+(1-F_1(w_1^*-\epsilon)))]}+ \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2))+(F_1(w_1^*)-F_2(w_1^*))))][\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2-\epsilon))+(F_1(w_1^*)-F_2(w_1^*-\epsilon)))]} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))][\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)+f_1(w_1^*))))]}+ \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))][\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)+f_1(w_1^*)))]}+ \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2)))][\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2))+f_1(w_1^*))]}
\end{aligned} \tag{11}
$$

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\begin{aligned}
\pi_2(\tilde{w}_2-\epsilon) &= \frac{\lambda^s\delta^s n^s}{D^s(\tilde{w}_k-\epsilon)D^s(\tilde{w}_k-2\epsilon)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(\tilde{w}_2-\epsilon)D^{0a}(\tilde{w}_2-2\epsilon)} + \frac{\lambda^0\delta^0 n^{0m}}{D^{0m}(\tilde{w}_2-\epsilon)D^{0m}(\tilde{w}_2-2\epsilon)} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2-\epsilon))+(1-F_1(w_1^*-\epsilon)))][\delta^s+\lambda^s((1-F_2(\tilde{w}_2-2\epsilon))+(1-F_1(w_1^*-2\epsilon)))]}+ \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2-\epsilon))+(1-F_1(w_1^*-\epsilon)))][\delta^0+\lambda^0((1-F_2(\tilde{w}_2-2\epsilon))+(1-F_1(w_1^*-2\epsilon)))]}+ \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2-\epsilon))+(F_1(w_1^*)-F_2(w_1^*-\epsilon)))][\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2-2\epsilon))+(F_1(w_1^*)-F_2(w_1^*-2\epsilon)))]} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)+f_1(w_1^*)))]^2}+ \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)+f_1(w_1^*)))]^2}+ \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2))+f_1(w_1^*))]^2}
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
\pi_2(\tilde{w}_2+\epsilon) &= \frac{\lambda^s\delta^s n^s}{D^s(\tilde{w}_2+\epsilon)D^s(\tilde{w}_2)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(\tilde{w}_2+\epsilon)D^{0a}(\tilde{w}_2)} + \frac{\lambda^0\delta^0 n^{0m}}{D^{0m}(\tilde{w}_2+\epsilon)D^{0m}(\tilde{w}_2)} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2+\epsilon))+(1-F_1(w_1^*+\epsilon)))][\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]}+ \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2+\epsilon))+(1-F_1(w_1^*+\epsilon)))][\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]}+ \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2+\epsilon)))][\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2)))]} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]^2}+ \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]^2}+ \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2)))]^2}
\end{aligned} \tag{13}
$$

As $f_1(w_1^*) > 0$ and $\epsilon \to 0$, it holds that $\pi_2(\tilde{w}_2-\epsilon) < \pi_2(\tilde{w}_2) < \pi_2(\tilde{w}_2+\epsilon)$. This implies that there will be no wage offers of value $\tilde{w}_2$. As $(ph - wh)$ increases with decreasing w, there might be a wage rate w' where it holds that $\pi_2(w') = \pi_2(\tilde{w}_2+\epsilon)$. This implies that

$f_2(.)$ exhibits a gap in the interval (w',$\tilde{w}_2$). If $\tilde{w}_2 < w_2^*$, the terms in equations 11 and 13 referring to workers of type $0m$ drop out. Although this might reduce the extent of the gap, $\pi_2(\tilde{w}_2) < \pi_2(\tilde{w}_2 + \epsilon)$ still holds. If $\tilde{w}_2 = w_2^*$ the necessary size of the mass point at w_2^* to balance the loss of type-$0m$ workers decreases (in comparison to $\tilde{w}_2 \neq w_2^*$). How large the gap is, i.e. whether any offers will be made below $\tilde{w}_2$ will depend on the economic environment captured by the parameters of the model.

2 Simulation results for the stylized version of the model

In the stylized version of the model presented in Appendix (A) of the main text, a version of the model is presented. This online appendix provides simulation results of that model.

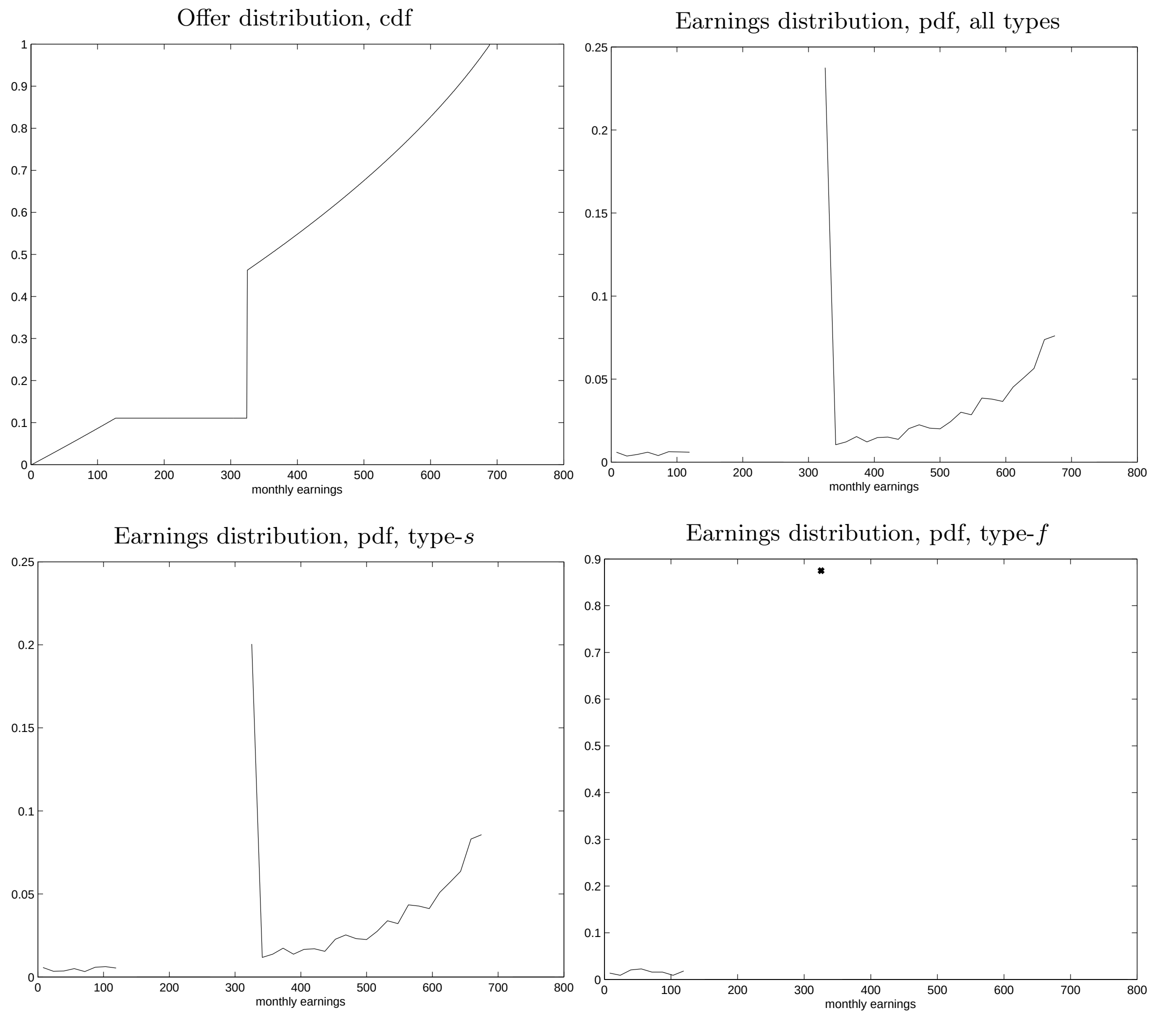
We simulate the model with the following parameter values: $p = 800$ €; $z^* = 325$ €; $\lambda^s = 0.2$; $\lambda^0 = 0.2$; $\delta = 0.1$; $n^s = 1$; $n^0 = 0.1$; $z^r = 0$.

The top-right panel of figure 1 shows a simulated earnings distribution based on the stylized model of the working paper:

Earnings above the minijob earnings threshold (325 €) increase smoothly on the top-left panel up to a maximum of $\bar{z} = 689$ € in our simulation. At the minijob threshold z^*=325 € there is a large mass point, following proposition (I). There is a gap below the threshold in line with proposition (II). Firms do not offer any earnings within the interval (127€- 325€). The additional margin of reducing offered earnings does not compensate for the discontinuously lower firm size. The resulting equilibrium cumulative offer distribution the top-left panel of figure 1.

The corresponding earnings distributions of the two types of workers are clearly influenced by the job offer distribution (figure 1): Although type-s workers have no tax incentive to bunch at z^*, the earnings distributions of both types exhibit a mass point here (for type-f workers the mass point is more prominent and - since these workers accept no job offers above the minijob level - there is no mass above). The upper right panel of figure 1 plots the resulting joint earnings distribution.

Figure 1: Offer and earnings distribution by types of workers



Notes: Type-s workers have or seek a small job as second job. Type-f workers have or seek a small job but have no other job. The minijob threshold is at 325€/month. While the cdf is calculated analytically, the pdf-graphs are based on 10000 drawings from the cdf. We simulated the model with the following parameter values: $p = 800$ €; $z^* = 325$ €; $\lambda^s = 0.2$; $\lambda^0 = 0.2$; $\delta = 0.1$; $n^s = 1$; $n^0 = 0.1$; $z^r = 0$.

References

Shephard, Andrew, "Equilibrium Search and Tax Credit Reform," *International Economic Review*, 2017, *58* (4).