Online Appendix
Equilibrium effects of making work pay policies: Evidence from Germany

Luke Haywood & Michael Neumann

14th April 2019

1 Discrete variation in working hours

In the main text, we assume continuous variation in hours worked. We here discuss the model equilibrium with discrete variation in hours: Weekly hours worked are in one of two categories h_k, such that $k \in (1, 2)$. In the market for low-paid jobs, this may correspond to 10 and 20 hours of working. We later consider how this model generalizes to three or more hours categories.

Firms set wage rates, w, workers derive utility from consumption and leisure. In order to simplify notation, we follow Shephard (2017) and define $q_2(w) = w$ and $U(q_1(w), h_1) = U(q_2(w), h_2) = U(w, h_2)$, so $q_1(w)$ is a function that denotes the wage rate that makes individuals indifferent between working with few ($k = 1$) hours at $q_1(w)$ or working more ($k = 2$) hours at w. Depending on preferences, individuals may require a low-hours wage premium or accept a low-hours wage penalty.

1.1 Worker mobility

The flow into and out of small jobs must balance (see appendix A of the main text). The flow of workers of type $j \in (s, 0a, 0m)$ from and into jobs with hours h_k and wage rate w is

$$D^j(w) g_k^j(q_k(w)) e_k^j = \lambda^j f_k(q_k(w)) \left(u^j + G_1^j(q_1(w) - \epsilon) e_1^j + G_2^j(w - \epsilon) e_2^j \right) \tag{1}$$

with $D^j(w) = [\delta + \lambda^j((1 - F_2(w)) + (1 - F_1(q_1(w))))]$ for $j \in (s, 0a)$. Equation 2 states the corresponding definition for workers of type $0m$ who do not accept jobs with wage rates larger than w_k^*.

$$D^{0m}(w) = \begin{cases} [\delta^0 + \lambda^0((F_2(w_2^*) - F_2(w)) + (F_1(w_1^*) - F_1(q_1(w))))] & \forall w \leq w_2^* \text{ and } q_1(w) \leq w_1^* \\ [\delta^0 + \lambda^0(F_2(w_2^*) - F_2(w))] & \forall w \leq w_2^* \text{ and } q_1(w) > w_1^* \end{cases} \quad (2)$$

The LHS of equation 1 pertains to workers who leave a job in a sector k with wage $q_k(w)$. For $k = 2$ this group consists of workers who move from sector 2 to sector 1 ($\lambda^j(1 - F_1(q_1(w)))g_2^j(w)e_2^j$ for $j \in (s, 0a)$), who move to a better paying job within sector 2 ($\lambda^j(1 - F_2(w))g_2^j(w)e_2^j$ for $j \in (s, 0a)$) and who lose their small job ($\delta^j g_2^j(w)e_2^j$). The RHS pertains to workers who start a job in sector k with wage rate $q_k(w)$. For $k = 2$ this consists of workers who move from sector 1 to sector 2 ($\lambda^j f_2(w)G_1^j(q_1(w) - \epsilon)e_1^j$), who changes jobs within sector 2 ($\lambda^j f_2(w)G_2^j(w - \epsilon)e_2^j$) and who were previously job-seeking ($\lambda^j f_2(w)$). The overall flow (i.e. both sectors) due to separations from jobs with wage rate of no greater than w is:

$$\begin{aligned} &(G_1^j(q_1(w))e_1^j + G_2^j(w)e_2^j)D^j(w) = \lambda^j u^j F_1(q_1(w)) + \lambda^j u^j F_2(w) \\ &= \lambda^j u^j + \lambda^j u^j - \lambda^j u^j(1 - F_1(q_1(w))) - \lambda^j u^j(1 - F_2(w)) \end{aligned} \quad (3)$$

Using equation **??** gives

$$G_1^j(q_1(w))e_1^j + G_2^j(w)e_2^j = \frac{\delta^j n^j - u^j D^j(w)}{D^j(w)}. \quad (4)$$

By combining equations 1 and 4 we obtain

$$g_k^j(q_k(w))e_k^j = \frac{\lambda^j f_k(q_k(w)) \left[u^j + \frac{\delta^j n^j - u^j(D^j(w-\epsilon))}{D^j(w-\epsilon)}\right]}{D^j(w)} \quad (5)$$

1.2 Firm size

The number of workers of type j in steady-state employed at a firm in sector k which offers wage rate $q_k(w)$ is

$$\begin{aligned} l_k^j(q_k(w)) &= \frac{g_k^j(q_k(w))e_k^j}{f_k(q_k(w))} \\ &= \frac{\lambda^j \delta n^j}{D^j(w)D^j(w - \epsilon)}. \end{aligned} \quad (6)$$

The steady state firm size is then

$$\begin{aligned} l_k(q_k(w)) &= l_k^s(q_k(w)) + l_k^{0a}(q_k(w)) + l_k^{0m}(q_k(w)) \\ &= \begin{cases} \frac{\lambda^s \delta^s n^s}{D^s(w)D^s(w-\epsilon)} + \frac{\lambda^0 \delta^0 n^{0a}}{D^{0a}(w)D^{0a}(w-\epsilon)} + \frac{\lambda^0 \delta^0 n^{0m}}{D^{0m}(w)D^{0m}(w-\epsilon)} & \forall w \leq w^* \\ \frac{\lambda^s \delta^s n^s}{D^s(w)D^s(w-\epsilon)} + \frac{\lambda^0 \delta^0 n^{0a}}{D^{0a}(w)D^{0a}(w-\epsilon)} & \forall w > w^* \end{cases}. \end{aligned} \quad (7)$$

Following the standard arguments of profit equalization, we find the following (the reasoning is parallel to the case without hours variation):

Proposition (A1) *There can be (at most one) mass point in the wage offer distribution at the threshold in each sector, i.e. at wages* $w_k^* \equiv \frac{z^*}{h_k}$.

Sketch of Proof: The following argument closely mirrors the argument in the case of homogeneous hours. We compare profits at the threshold value with profits above. We find that if there exist offers above, there must be a mass point at the threshold.

The profit of a sector k firm offering wage rate $q_k(w)$ can be expressed as $\pi_k(q_k(w)) = (ph_k - q_k(w)h_k)l_k(q_k(w))$. We first state the profits of a type-2-firm, assuming that $q_1(w_2^*) \leq w_1^*$.

$$\begin{aligned}\pi_2(w_2^*) &= \frac{\lambda^s\delta^s n^s}{D^s(w_2^*)D^s(w_2^*-\epsilon)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(w_2^*)D^{0a}(w_2^*-\epsilon)} + \frac{\lambda^0\delta^0 n^{0m}}{D^{0m}(w_2^*)D^{0m}(w_2^*-\epsilon)} \\ &= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(w_2^*))+(1-F_1(q_1(w_2^*))))][\delta^s+\lambda^s((1-F_2(w_2^*-\epsilon))+(1-F_1(q_1(w_2^*)-\epsilon)))]} + \\ &+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(w_2^*))+(1-F_1(q_1(w_2^*))))][\delta^0+\lambda^0((1-F_2(w_2^*-\epsilon))+(1-F_1(q_1(w_2^*)-\epsilon)))]} + \\ &+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0(F_1(w_1^*)-F_1(q_1(w_2^*)))][\delta^0+\lambda^0(f_2(w_2^*)+(F_1(w_1^*)-F_1(q_1(w_2^*)-\epsilon)))]}\end{aligned} \tag{8}$$

Evaluated marginally above the threshold, profits are

$$\begin{aligned}\pi_2(w_2^*+\epsilon) &= \frac{\lambda^s\delta^s n^s}{D^s(w_2^*+\epsilon)D^s(w_2^*)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(w_2^*+\epsilon)D^{0a}(w_2^*)} \\ &= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(w_2^*+\epsilon))+(1-F_1(q_1(w_2^*+\epsilon))))][\delta^s+\lambda^s((1-F_2(w_2^*))+(1-F_1(q_1(w_2^*))))]} + \\ &+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(w_2^*+\epsilon))+(1-F_1(q_1(w_2^*+\epsilon))))][\delta^0+\lambda^0((1-F_2(w_2^*))+(1-F_1(q_1(w_2^*))))]}\end{aligned} \tag{9}$$

Equations 8 and 9 show that the equal profit condition can only hold if there is a mass point in the offer distribution of sector 2 at w_2^*. By symmetry, note that the same argument can be made with respect to a type-1 firm. However, if the utility of a threshold offer lies in the "gap area" due to a threshold in another sector, it may be the case that there is no mass point in that sector. This explains the restriction "at most one" in Proposition (IV) and completes our discussion.

We now consider the influence of thresholds in other hours sectors on the wage distribution. Consider a firm of type 2, i.e. seeking a worker to work for h_2 hours. The impact of a potential mass point in the offer distribution of sector 1 at w_1^* depends on the relation between w_2^*, $q_1(w_2^*)$ and w_1^*.

Proposition (A2) *There will be no wage offers at wage levels (and in a certain interval below this level) that offer the same utility as is available at threshold wages $w_{j\neq k}^*$ in other sectors.*

The intuition for Proposition (A2) is the following: It is a dominated strategy to offer a wage rate that is equal in utility to an offer made by several other firms. A slightly higher offer will attract all workers from these firms at only marginal cost. By Proposition (IV), wage offers at earnings thresholds generate mass points in the wage offer distributions. Thus for example a type-2 firm will offer a wage rate slightly larger than $\tilde{w}_2$ (where $U(\tilde{w}_2, h_2) = U(w_1^*, h_1)$.) in order to additionally attracts workers from this positive mass of sector 1 firms. This implies that there must be a gap in the wage offer distribution at $\tilde{w}_2$. How much below this utility value an offer can be sustained in equilibrium will depend on the parameters of the model in an analogous way to the potential existence of offers below the threshold offer in the homogeneous case.

Sketch of proof: Let $\tilde{w}_2$ denote the wage rate which satisfies $U(\tilde{w}_2, h_2) = U(w_1^*, h_1)$. If $\tilde{w}_2 > w_2^*$ the profits of a sector 2 firm offering wage rate $\tilde{w}_2$ and slightly above are:

$$
\begin{aligned}
\pi_2(\tilde{w}_2) &= \frac{\lambda^s \delta n^s}{D^s(\tilde{w}_2) D^s(\tilde{w}_2 - \epsilon)} + \frac{\lambda^0 \delta^0 n^{0a}}{D^{0a}(\tilde{w}_2) D^{0a}(\tilde{w}_2 - \epsilon)} + \frac{\lambda^0 \delta^0 n^{0m}}{D^{0m}(\tilde{w}_2) D^{0m}(\tilde{w}_2 - \epsilon)} \\
&= \frac{\lambda^s \delta^s n^s}{[\delta^s + \lambda^s((1 - F_2(\tilde{w}_2)) + (1 - F_1(w_1^*)))][\delta^s + \lambda^s((1 - F_2(\tilde{w}_2 - \epsilon)) + (1 - F_1(w_1^* - \epsilon)))]} + \\
&+ \frac{\lambda^0 \delta^0 n^{0a}}{[\delta^0 + \lambda^0((1 - F_2(\tilde{w}_2)) + (1 - F_1(w_1^*)))][\delta^0 + \lambda^0((1 - F_2(\tilde{w}_2 - \epsilon)) + (1 - F_1(w_1^* - \epsilon)))]} + \\
&+ \frac{\lambda^0 \delta^0 n^{0m}}{[\delta^0 + \lambda^0((F_2(w_2^*) - F_2(\tilde{w}_2)) + (F_1(w_1^*) - F_2(w_1^*))))][\delta^0 + \lambda^0((F_2(w_2^*) - F_2(\tilde{w}_2 - \epsilon)) + (F_1(w_1^*) - F_2(w_1^* - \epsilon)))]} \\
&= \frac{\lambda^s \delta^s n^s}{[\delta^s + \lambda^s((1 - F_2(\tilde{w}_2)) + (1 - F_1(w_1^*)))][\delta^s + \lambda^s((1 - F_2(\tilde{w}_2)) + (1 - F_1(w_1^*) + f_1(w_1^*))))]} + \\
&+ \frac{\lambda^0 \delta^0 n^{0a}}{[\delta^0 + \lambda^0((1 - F_2(\tilde{w}_2)) + (1 - F_1(w_1^*)))][\delta^0 + \lambda^0((1 - F_2(\tilde{w}_2)) + (1 - F_1(w_1^*) + f_1(w_1^*)))]} + \\
&+ \frac{\lambda^0 \delta^0 n^{0m}}{[\delta^0 + \lambda^0((F_2(w_2^*) - F_2(\tilde{w}_2)))][\delta^0 + \lambda^0((F_2(w_2^*) - F_2(\tilde{w}_2)) + f_1(w_1^*))]}
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
\pi_2(\tilde{w}_2-\epsilon) &= \frac{\lambda^s\delta^s n^s}{D^s(\tilde{w}_k-\epsilon)D^s(\tilde{w}_k-2\epsilon)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(\tilde{w}_2-\epsilon)D^{0a}(\tilde{w}_2-2\epsilon)} + \frac{\lambda^0\delta^0 n^{0m}}{D^{0m}(\tilde{w}_2-\epsilon)D^{0m}(\tilde{w}_2-2\epsilon)} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2-\epsilon))+(1-F_1(w_1^*-\epsilon)))][\delta^s+\lambda^s((1-F_2(\tilde{w}_2-2\epsilon))+(1-F_1(w_1^*-2\epsilon)))]}+ \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2-\epsilon))+(1-F_1(w_1^*-\epsilon)))][\delta^0+\lambda^0((1-F_2(\tilde{w}_2-2\epsilon))+(1-F_1(w_1^*-2\epsilon)))]}+ \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2-\epsilon))+(F_1(w_1^*)-F_2(w_1^*-\epsilon)))][\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2-2\epsilon))+(F_1(w_1^*)-F_2(w_1^*-2\epsilon)))]} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)+f_1(w_1^*)))]^2}+ \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)+f_1(w_1^*)))]^2}+ \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2))+f_1(w_1^*))]^2} \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
\pi_2(\tilde{w}_2+\epsilon) &= \frac{\lambda^s\delta^s n^s}{D^s(\tilde{w}_2+\epsilon)D^s(\tilde{w}_2)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(\tilde{w}_2+\epsilon)D^{0a}(\tilde{w}_2)} + \frac{\lambda^0\delta^0 n^{0m}}{D^{0m}(\tilde{w}_2+\epsilon)D^{0m}(\tilde{w}_2)} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2+\epsilon))+(1-F_1(w_1^*+\epsilon)))][\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]}+ \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2+\epsilon))+(1-F_1(w_1^*+\epsilon)))][\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]}+ \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2+\epsilon)))][\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2)))]} \qquad (12) \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]^2}+ \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]^2}+ \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2)))]^2}
\end{aligned}
$$

As $f_1(w_1^*) > 0$ and $\epsilon \to 0$, it holds that $\pi_2(\tilde{w}_2-\epsilon) < \pi_2(\tilde{w}_2) < \pi_2(\tilde{w}_2+\epsilon)$. This implies that there will be no wage offers of value $\tilde{w}_2$. As $(ph - wh)$ increases with decreasing w, there might be a wage rate w' where it holds that $\pi_2(w') = \pi_2(\tilde{w}_2+\epsilon)$. This implies that $f_2(.)$ exhibits a gap in the interval (w',$\tilde{w}_2$). If $\tilde{w}_2 < w_2^*$, the terms in equations 10 and 12 referring to workers of type $0m$ drop out. Although this might reduce the extent of the gap, $\pi_2(\tilde{w}_2) < \pi_2(\tilde{w}_2+\epsilon)$ still holds. If $\tilde{w}_2 = w_2^*$ the necessary size of the mass point at w_2^* to balance the loss of type-$0m$ workers decreases (in comparison to $\tilde{w}_2 \neq w_2^*$). How large the gap is, i.e. whether any offers will be made below $\tilde{w}_2$ will depend on the economic environment captured by the parameters of the model.

References

Shephard, Andrew, "Equilibrium Search and Tax Credit Reform," *International Economic Review*, 2017, *forthcoming.*