# Equilibrium effects of making work pay policies: Evidence from Germany

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**Abstract**

Making work pay (MWP) policies are intended to increase participation of workers with low earnings. However, they may also affect other workers. Estimating welfare consequences of MWP policies therefore requires equilibrium models of the labour market. We show this using a tax exemption that applied to one group of workers, but not another. This exemption strongly affects both groups' earnings distribution. To rationalize this result we present the first equilibrium job-search model including discontinuous taxes. Our model includes heterogeneity of both workers and firms and can rationalize the observed earning distributions across different types of minijob workers. Using German administrative data we estimate equilibrium effects of the minijob tax exemption on participation, hours and wages. Results highlight that equilibrium effects resulting from firm responses increase distributional effects of tax exemptions: Workers who would work anyway are hurt by subsidies benefiting groups who enter the market as a result of tax incentives.

**Keywords**: Tax exemptions, Welfare-to-work, Labor Demand, Job search, Firm responses, Bunching.

**JEL Codes**: J64 ; J31 ; J22 ; J23

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# 1 Introduction

The expansion of welfare-to-work or making-work-pay (MWP) programmes has come with increasing awareness of potential negative side effects (e.g. Bargain et al. (2010)). MWP policies typically reward workers with low earnings (e.g. the Earned Income Tax Credit in the US or the Working Family Tax Credit in the UK, see Meyer and Rosenbaum (2001) for the US, Blundell (2000) for the UK and Bargain and Orsini (2006) for continental Europe). In the case of Germany, jobs with earnings up to 325 euros, the so-called minijobs, are exempt from income tax and social security contributions (SSC). Side effects of such a MWP policies can be split into two components. First, firms and workers might react to incentives built into the policy. Reducing hours of work and/or wages could, for example, make workers eligible for the policy. Second, equilibrium effects affect individuals beyond those directly targetted by the policy. For example, MWP policies may induce firms to change the distribution of jobs and wages. Chetty et al. (2011) show that if firms cannot easily change the hours of a posted job offer, they have an incentive to package their hours-wage bundles according to average preferences of job-seeking workers. Since the same jobs were eligible for tax reduction for one group but not another, identification of the role of firm responses is particularly intuitive. This paper sets up and empirically estimates an equilibrium job search model with a discontinuous tax schedule in order to model both extensive and intensive margin reactions to the German minijob policy in equilibrium.

The German setting is well-suited for an empirical application of our equilibrium job search model: Throughout the period of analysis (1999-2002), tax exemptions in Germany were awarded to workers only if workers did not hold a second job. Employees who held a small job as a second job were not covered by the policy at all. While they are found to react less strongly to the policy than directly affected workers, we find indisputable bunching at the tax exemption threshold for both groups. The evidence of an equilibrium effect for workers whose budget set is unaffected allows us to identify the two effects of tax subsidies: For workers not subject to the minijob tax exemption, changes in hours and wages result only due to firms' responses. We estimate our model using a combination of German administrative and household survey data. We find that the tax and SSC exemption for low earnings provoke substantial distortions. With our estimated structural model we then simulate different counter-factual policies. We show that distortions can be removed by smoothing the tax schedule. These simulations highlight the indirect costs of (discontinuous) tax policies.

We make two main contributions. First, our unique institutional setting allows for a particularly convincing identification of equilibrium effects. Tax subsidies offered to certain workers with low earnings end up affecting many other workers in the labor market. We estimate our equilibrium job search model and quantify these distorting effects. We thereby complement work discussing unintended equilibrium consequences of MWP policies (Rothstein, 2010; Leigh, 2010; Kolm and Tonin, 2011) as well as the firms' role in responses to tax reforms in general (Kopczuk and Slemrod, 2006) and bunching in particular (Best, 2014; Gudgeon and Trenkle, 2017; Chetty et al., 2011). In particular, we complement Tazhitdinova (2017) who identifies lower wage rates and fringe benefits for German minijob workers as channels by which firms reinforce bunching at the minijob threshold. Furthermore, she proposes using the observed bunching to estimate labour supply elasticities. This approach focuses on hours responses of individuals who *reduce* their earnings due to the tax exemption. It does not allow an estimation of the increase in employment up to the minijob threshold - this participation margin is a very important for policy, however. We structurally estimate the relative importance of participation and hours margins, and find participation effects to be large, with over one third of minijobs held by individuals who would have zero earnings (rather than higher earnings) without the tax exemption. Second, we include discontinuous taxes into a standard equilibrium job search model (Shephard (2017) includes continuous taxes). In line with the bunching literature (Saez, 2010), we believe important additional welfare costs are caused by the discontinuous nature of taxes. We show that these are further increased by firm responses. In contrast to previous equilibrium job search models, we are able to rationalize discontinuous bunching at tax thresholds in the interior of the earnings distribution. We are thus able to analyze counter-factual policy reforms that aim at removing the welfare costs induced by tax non-linearities.

Our model is related to evaluations of MWP policies based on analyses of labor supply as a discrete choice (specifically for minijobs, see Arntz et al. (2003) and Bargain et al. (2010)). Given that they lack an equilibrium framework, the wage distribution is typically taken as given. The evidence of equilibrium responses is a strong indication that individuals cannot freely choose hours, however. The models most similar to ours thus include restrictions on the free choice of hours (e.g. Van Soest et al. (1990), Bloemen (2008), Beffy et al. (2015)). Job search models provide a way of explaining some of these constraints without resorting to ad-hoc restrictions on the set of labor supply choices available to individuals.

A different strand of the literature has investigated the reason for the divergence between labor supply elasticities based on individual-level data from that based on more aggregate data. Chetty et al. (2011) argue that firm responses may be a key to explaining "micro versus macro elasticities", but refrain from estimating a structural search model, which allows a welfare analysis of tax policy in the light of firm responses. The difference between macro and micro labor supply elasticities can be understood in our model: Assume there is a change in labor taxes. The micro elasticity corresponds to an individuals' labor supply reaction to changed net wages for a given (gross) wage offer distribution. The macro supply elasticity gives changes in volume of work after taking into account firms' changes to the gross wage distribution as a result of the tax change.

Section 2 outlines the characteristics of the market for small jobs in Germany. Section 3 presents our model and simulation results. Section 4 provides information on our data and descriptive statistics. Section 5 discusses identification and estimation. Section 6 presents estimation results and an application of the model. Section 7 concludes.

## 2 Minijobs in Germany

This study focuses on regular low-paid employment[1], a sector subject to special tax and social security treatment. We focus on the institutions as they existed from 1999-2003. The distortion we focus on continues to exist to this day, but identification is not as clean after a large reform in 2003, since all small jobs are now covered, regardless of whether they workers' other earnings.

Employment contracts qualify for special tax and SSC treatment if monthly earnings do not exceed 325 € and jobs have fewer than 15 hours per week.[2] These employment relations are designated as minijobs and are not subject to employee SSC[3]. First, crossing the threshold results in a decrease of net earnings because when an individual earns more

[1]We use *small jobs* employment to describe jobs with low earnings. This covers all employment relations that yield regular earnings below 800 € per month. We exclude short-term employment consisting of jobs that last for less than a certain number of days per year as they are subject to different rules.

[2]The monthly threshold may be exceeded for two months within a year as long as monthly earnings are on average below the threshold in that year.

[3]Employers' SSC are 10% (health insurance) and 12% (pension contributions) of minijob earnings and thus approximately correspond to employers' SSC for other employment contracts.

than the threshold, her entire earnings are subject to SSC (around 21% of gross earnings), i.e. the budget set features a notch[4] as well as a kink. Whilst minijobs do not create an entitlement to social security (and jobs with earnings above do), we do not believe that entitlement effects for either health or pension insurance are important in the market for low-paid employment: Firstly, survey data shows that individuals who only hold minijobs are mainly housewives/-men, students, pensioners and registered job-seekers (Körner et al., 2013). In Germany almost all individuals in these groups are either health-insured through their partners, the state or have access to specific student health care provision. Secondly, pension contributions from low-paid employment would not suffice to increase pension benefits above the social assistance level which is available independent of contributions. We thus treat SSC in this market as a pure tax.

Second, in addition to being exempt from SSC, minijob earnings are not liable to income tax. Since earnings above this level were subject to the income tax, the size of the additional tax notch depends on the other factors influencing a person's income tax above the minijob notch (in particular, spousal earnings). Since the general income tax allowance was above the minijob threshold, there is no additional income tax notch for singles with no other income. The notch can be very large for married individuals, though. The size of the drop in net earnings is thus composed of SSC and income taxation and varies across individuals. As a reaction to this notch, workers who only hold a small job (designated here as *type-0 workers*) strongly bunch at the minijob threshold (figure 1, left panel).

Importantly for this study, if the worker has another job subject to SSC on top of a low-paid job, the latter cannot qualify as a minijob[5] and is therefore subject to full SSC and income taxation. There is thus no incentive to bunch at the minijob threshold. Nevertheless, figure 1 reveals strong bunching of earnings for workers who also have full time[6] jobs (in the following referred to as *type-s workers*). To explain this phenomenon, we set up an equilibrium model in the next section including firms' wage-setting and workers'

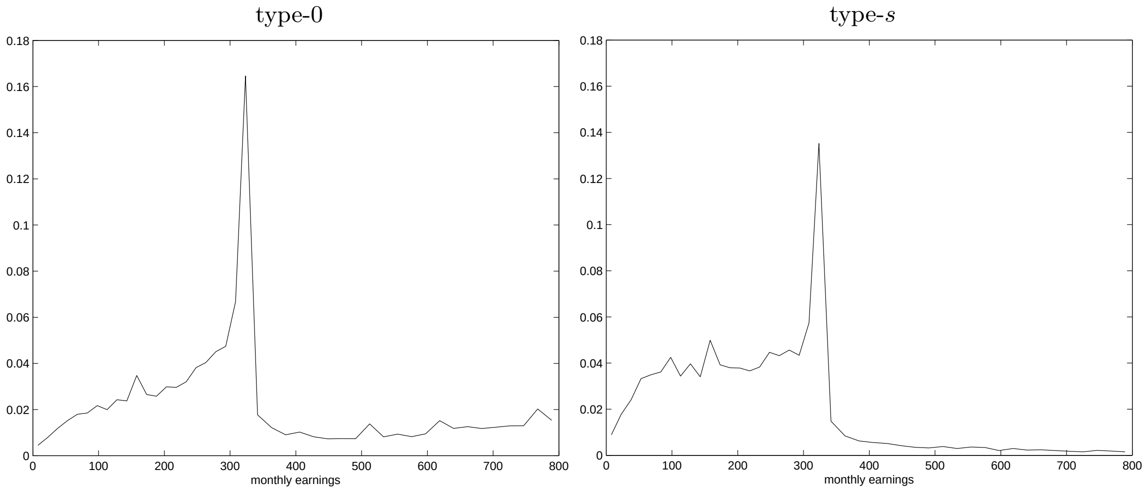
[4] In March 2002, a policy called *Mainzer Modell* aimed at improving incentives by balancing the additional SSC due to crossing the threshold by a subsidy which was gradually withdrawn for higher household earnings. The notch with respect to income taxation remained. We nevertheless limit the empirical analysis to the period before March 2002, see section 4.

[5] Unless joint income of two minijobs did not exceed the threshold, a rare case in our data.

[6] We use the term "full time" to fix ideas and as over 80% of these first jobs of second-job holders are full-time. The remaining are part-time, see table 5.

job search.

**Figure 1:** Distribution of observed earnings by types of workers (Note: Type-$s$ have no incentive to bunch)




*Notes:* Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0 workers have or seek a low-paid job and do not have another job. The minijob threshold is at 325€/month.
*Source:* SIAB.

# 3 The Model

Equilibrium effects of a MWP policy arise when firms cannot easily adjust their job offers to individual job-seekers with different preferences or tax treatment. Firms then have an incentive to tailor their job offers to the expected preferences of job-seekers. We model this effect using an equilibrium search model with wage posting. First, job search models provide a convenient way of modeling the limited labor supply options most people face: job offers need to be sought and, once found, typically come with fixed hours. Second, by contrast to simple search models, equilibrium models endogenize the job offer distribution. If earnings could be adjusted costlessly (in a simple bargaining framework), we should expect no bunching at thresholds that do not apply to individuals personally. The data suggests otherwise, leading us to assume that firms make take-it-or-leave-it offers along the lines of Burdett and Mortensen (1998) and Bontemps et al. (1999).[7] The endogenous job offer distribution then reflects the preference distribution of the whole population of job-seekers.

We are mainly interested in the sector for low-paid employment and assume that

[7]An alternative modeling choice would be to allow for negotiations for a fraction of workers along the lines of Chetty et al. (2011). In the market for small jobs, our assumption of take-it-or-leave-it offers appears particularly credible, see (Shephard, 2017).

these jobs form a separate labor market. Section 4.3 shows that the industry sector distributions of minijobs and full-time jobs are very different and that the former are mainly concentrated in cleaning, security and other firm services, as well as retail. The focus on the low-paid market implies that first jobs (full-time jobs) of second job holders are not modeled but treated as constant and exogenous. In other words, these jobs importantly determine workers' experience in the small jobs market (we allow for different rates of job offer arrivals and separations). However, small jobs are not allowed to influence whether workers accept full-time jobs or not. This appears reasonable as the group of type-0 workers includes mainly pensioners, students and spouses of high earners not looking for a full-time job.

### 3.1 Model Basics

The small jobs market is composed of a continuum of workers and firms. Some workers have a full-time job but are nonetheless active in the market for small jobs, seeking a second job (type-$s$ workers of which there are $n^s$). Type-$s$ workers are not qualified for the tax exemption, which is only available for workers who have no other job - i.e. type-0 workers. Thus the budget set of type-0 workers exhibits a notch and a kink at gross earnings $z = z^*$ while it is smooth for type-$s$ workers. Omitting the individual index for simplicity, net earnings $c$ for workers of type $j$ are then given by

$$c = \begin{cases} z & \forall z \leq z^* \ \& \ j = 0 \\ z(1 - t^{ssc})(1 - t^{inc}) & \forall z > z^* \ \& \ j = 0 \\ z(1 - t^{ssc})(1 - t^{inc}) & \forall j = s, \end{cases} \tag{1}$$

where $t^{ssc}$ denotes employees' SSC rate and $t^{inc}$ the income tax rate. We assume utility $v$ depends on consumption and leisure $\bar{h} - h$ ($\bar{h}$ is total available hours which we set to 12 hours a day). $\alpha$ gives the elasticity of $v$ with respect to net earnings and leisure, such that we have $v = c^{\alpha}(\bar{h} - h)^{(1-\alpha)}$.

Workers receive small job offers both when they already have a small job (on-the-job search) and when they do not by drawing from a known job offer distribution $F(.)$. The exogenous job offer arrival rate ($\lambda$) is allowed to differ across workers with and without full-time jobs (types $s$ and 0 respectively), but is independent of whether or not a worker already has a second job. We test the robustness of our results with respect to this assumption by making two alternative assumptions in appendix G. Workers lose their job at an exogenous rate $\delta$ which we also allow to differ across worker types. Workers seek to maximize expected steady-state future utility $v$. Individuals seeking a small job accept job offers based on their individual tax treatment and only if the job exceeds the reservation utility $v^r$. Firms may freely enter the market and seek to maximize profits $\pi = [p-w]hl(v)$ where $p$ is turnover per working hour, $w = \frac{z}{h}$ the gross hourly wage rate and $l(v)$ the size of the labor force of a firm offering jobs with utility $v$ (each firm offers one type of job).

The remainder of this section first presents a stylized model with no variation in hours worked. In this scenario, we rationalize bunching in the interior of the earnings distribution (section 3.2). We then present a model with heterogeneous workers and firms. Specifically, small-job-seekers have different marginal tax rates and outside options - varying the attractiveness of tax-free minijobs as compared to higher-paying but taxable jobs (section 3.3). This second, more realistic, version of the model is brought to the data in section 5. In both cases firms' productivity is assumed to be homogeneous. However, we argue that heterogeneity in hours has similar implications when hours of work is not a choice variable of firms but predetermined by technology.[8]

### 3.2 Stylized Model

The objective of this section is to show how a notch in the budget set of one group of workers affects the earnings distribution of all workers. We can thereby rationalize bunching in the interior of the earnings distribution. Two simplifying assumptions enable us to solve the model analytically (both are relaxed in the next section). First, job offers contain no variation in hours. This implies that utility increases monotonously in net earnings, which thus completely describe a job offer. Second, we account for the different incentives that the tax system generates for the workers in our two sub-populations: Full-time workers seeking a second job (type-$s$) have no discontinuous incentive to locate at or below the threshold, since no tax deductions apply. In this first stylized model we assume that type-0 workers only accept job offers with earnings up to the threshold. This is consistent with very large differences in tax rates applicable above the minijob earnings: $t^{ssc} = t^{inc} = 1$ for type-0 workers and $t^{ssc} = t^{inc} = 0$ for type-$s$ worker (recall that whilst all type-0 workers have higher taxes above the threshold, there is variation concerning the marginal tax rate). Importantly, the prominence of type-0 workers generates a strong

[8]Firms have different possibilities of resorting to tax-free minijobs which we model as fixed hours requirements. Differentiating firms by hours requirements creates simple and continusous variation in the attractiveness of minijob contracts up 325€.

incentive for firms to over-proportionally offer such jobs. We now use information about worker mobility to establish the firm size distribution $l(z)$, critical in determining firms' optimal job offers in equilibrium.

**Worker mobility**

In equilibrium, the flows of workers of each type $j$ moving in and out of small jobs must balance.

$$\delta^j(n^j - u^j) = \begin{cases} \lambda^j u^j & \text{for } j = s \\ \lambda^j u^j F(z^*) & \text{for } j = 0 \end{cases} \tag{2}$$

where $u^j$ denotes the number of small-job-seeking type-$j$ workers and $\kappa^j = \frac{\lambda^j}{\delta^j}$. The flows differ across types since type-0 workers do not accept jobs with earnings $z > z^*$. The measures of small-job-seekers are thus:

$$u^j = \begin{cases} \frac{n^j}{1+\kappa^j} & \text{for } j = s \\ \frac{n^j}{1+\kappa^j F(z^*)} & \text{for } j = 0 \end{cases} \tag{3}$$

Similarly, in the steady-state the flow of small-job-seekers into small jobs with earnings no greater than $z$ must equal the measure of small job separations. The latter comprises employees losing their small job with value no greater than $z$ as well as workers moving to better small job (the left-hand side of equation 4 with $G^j(.)$ denoting the cumulative density function of realized earnings for type-$j$ workers).

$$\lambda^j F(z) u^j = \begin{cases} [\delta^j + \lambda^j(1 - F(z))]G^j(z)(n^j - u^j) & \text{for } j = s \\ [\delta^j + \lambda^j(F(z^*) - F(z))]G^j(z)(n^j - u^j) & \text{for } j = 0 \ \ \& \ \ z \leq \ z^* \end{cases} \tag{4}$$

**Firm size**

In the steady-state the number of workers of type $j$ employed at a firm offering jobs with earnings $z$ can be expressed by equation 5, which follows Burdett and Mortensen (1998).

$$l^j(z) = \lim_{\epsilon \to 0} \frac{(G^j(z) - G^j(z - \epsilon))(n^j - u^j)}{F(z) - F(z - \epsilon)} \quad \text{for } j \in (s, 0) \tag{5}$$

Using expressions 2, 4 and 5, appendix A shows that firm size is increasing in $z$ both below and above $z^*$, but with a discontinuity at $z^*$, since type-0 workers do not accept jobs above this threshold value:

$$l(z) = l^s(z) + l^0(z)$$
$$= \begin{cases} \frac{n^s\kappa^s}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))} + \frac{n^0\kappa^0}{(1+\kappa^0(F(z^*)-F(z)))(1+\kappa^0(F(z^*)-F(z-\epsilon)))} & \forall z \leq z^* \\ \frac{n^s\kappa^s}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))} & \forall z > z^* \end{cases} \tag{6}$$

**Equilibrium job offer distribution**

As firms are profit-maximizing and ex-ante identical, they must be indifferent about offering any $z$ offered in the market. Firms offering jobs with low earnings achieve higher profits per worker but attract fewer workers than firms offering jobs with higher earnings, as in Burdett and Mortensen (1998). However, when earnings exceed $z^*$, $n^0$ individuals drop out, creating a discontinuity. The endogenous offer distribution might, therefore, include a mass point because firms do not have an incentive to offer slightly higher earnings.

**Proposition (I)** *If we observe offers above $z^*$, there must be a mass point of job offers at $z^*$. The earnings offer distribution above $z^*$ is continuous up to the highest earnings offers, $\bar{z}$.*

**Proof:** See Appendix B.

The intuition of proposition (I) is that equal profits at and marginally above the threshold can only hold when the loss of type-0 workers is balanced by a discontinuously large number of type-$s$ workers that can be attracted by exceeding the threshold. This requires a mass point at $z^*$. An earnings offer slightly above the threshold is able to attract all type-$s$ workers currently earning $z^*$ (i.e. located at the mass point). As we observe positive mass above the threshold in the data (section 4, proposition (I) implies that there is a mass point at $z = z^*$ (i.e. that $f(z^*) > 0$).

**Proposition (II)** *If there is a mass point at $z^*$, there will be a gap in the offer distribution just below the threshold. There may or may not exist earnings offers below the threshold $z^*$ in equilibrium. If there are earnings below $z^*$, the earnings offer distribution will then be continuous between the left limit of the gap, $z''$, and reservation earnings, $z^r$.*

**Proof:** See Appendix B

To understand this proposition, note that a mass point in our setting implies that any job offer with earnings just below the mass point ($z^* - \varepsilon$) will earn less profits, since margins per worker are only slightly higher, but firm size will be discontinuously lower since there is a mass of firms (offering $z^*$) that can poach a worker employed at earnings $z^* - \varepsilon$. Given the lack of offers just below the mass point, i.e. a gap in the offer distribution, the question arises whether any earnings lower than $z^*$ are offered in equilibrium. This is the case when the increase in profit per worker is able to balance the lower firm size vis-à-vis locating at the mass point.

**Simulation**

The top-right panel of figure 2 shows the features of the earnings distribution simulated using our stylized model:[9] We observe that earnings above the minijob earnings threshold (325 €) increase smoothly on the top-left panel up to a maximum of $\bar{z} = 689$ € in our simulation. At the minijob threshold $z^*$=325 € there is a large mass point, following proposition (I). There is a gap below the threshold in line with proposition (II). Firms do not offer any earnings within that interval. The additional margin of reducing offered earnings compensates for the discontinuously lower firm size below the mass point not until a firm reduces its earnings to $z''$, which in our simulation is at 127 €. The resulting equilibrium cumulative offer distribution increases smoothly up to $z = z'' = 127$ and is constant in the interval $z \in [127, 325)$ (see the top-left panel of figure 2).

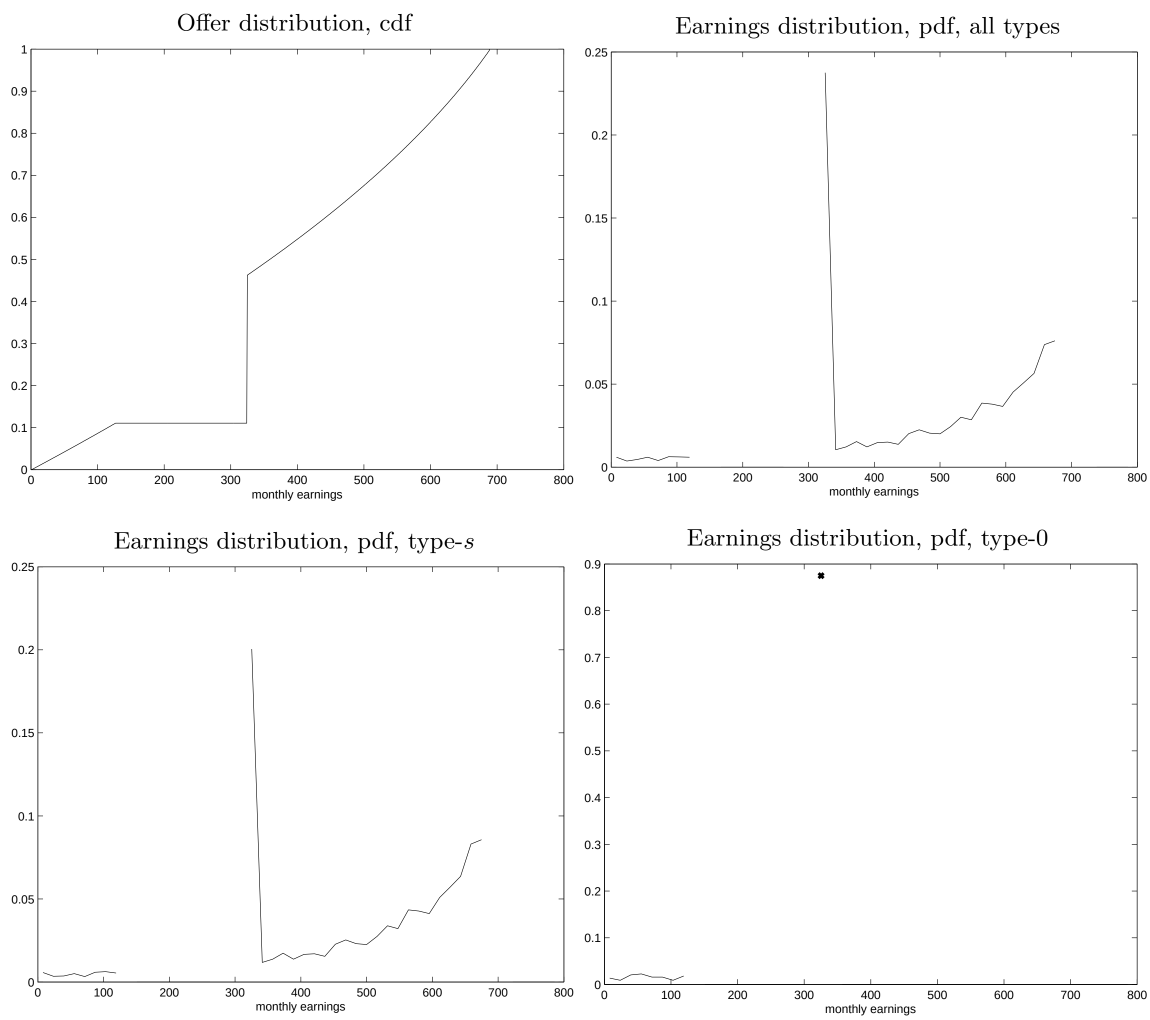
The corresponding earnings distributions of the two types of workers are clearly influenced by the job offer distribution (figure 2): Although type-$s$ workers have no tax incentive to bunch at $z^*$, the earnings distributions of both types exhibit a mass point here (for type-0 workers the mass point is more prominent and - since these workers accept no job offers above the minijob level - there is no mass above). The upper right panel of figure 2 plots the resulting joint earnings distribution.

## 3.3 Heterogeneous hours, taxes and worker heterogeneity

We now enrich the model with two important aspects which prevent an analytical solution: First, working hours differ. This is particularly relevant as the minijob threshold is based on monthly earnings, not hourly wages. Survey data show important variation in hours of low-paid workers (figure 3). This section thus builds an equilibrium model taking into account variation in hours. Including variation in working hours in our model allows both for the existence of earnings offers above the threshold and a lack of a gap below the

[9] We simulated the model with the following parameter values: $p = 800$ €; $z^* = 325$ €; $\lambda^s = 0.2$; $\lambda^0 = 0.2$; $\delta = 0.1$; $n^s = 1$; $n^0 = 0.1$; $z^r = 0$.

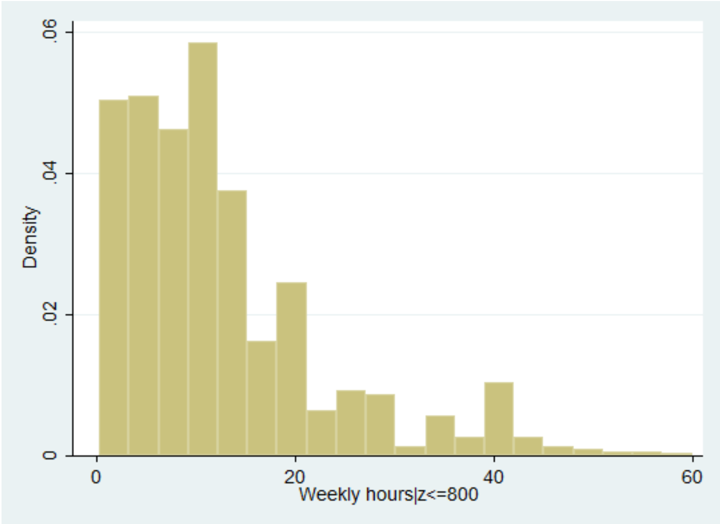
**Figure 2:** Offer and earnings distribution by types of workers




*Notes:* Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0 workers have or seek a low-paid job but have no other job. The minijob threshold is at 325€/month. While the cdf is calculated analytically, the pdf-graphs are based on 10000 drawings from the cdf. We simulated the model with the following parameter values: $p = 800$ €; $z^* = 325$ €; $\lambda^s = 0.2$; $\lambda^0 = 0.2$; $\delta = 0.1$; $n^s = 1$; $n^0 = 0.1$; $z^r = 0$.
*Source:* own calculations.

threshold. Requirements for some workers to work longer hours can also explain why not even more firms make use of minijobs. Second, although the small job earnings of type-0 workers (those who have no other job) bunch very strongly at the threshold, we observe a positive mass of first jobs above the threshold, contrary to our assumption of prohibitive taxation above the threshold for these workers. This section thus allows for individual variation in tax and SSC rates for $t^{inc}$ and $t^{ssc}$ in equation 1. This generates variation in incentives for type-0 workers to accept jobs above the threshold. Additionally, we include further variation in workers' willingness to accept only a minijob compared to a job with higher earnings. We now discuss these new aspects in turn.

**Figure 3:** Hours distribution - less than 800 €/month




*Notes:* Sample consists of jobs with earnings of less than 800 €/month

*Source:* GSOEP wave 2001

### Variance in working hours

We assume that hours vary across firms and workers care about hours and earnings. Following Chetty et al. (2011) we assume that hours of work are not a choice variable of firms but predetermined by technology. We further assume that the required hours of work vary continuously across firms.[10] Equal wage offers of different firms will thus correspond to different utility levels.[11] Our model of different hours requirement has predictions similar to including heterogeneity in productivity (see Bontemps et al. (1999)). The model implies that there is an optimal wage level yielding the largest utility for every hours requirement. Different firms have different optimal wage levels and offer different utility-levels. The attractiveness of a firm (to job-seekers) correlates positively with firms' profits. The ordering of firms depends on the preference parameter $\alpha$. As in models where attractiveness of firms varies as a result of heterogeneity in productivity, firms follow pure

[10] Allowing for discrete variation in hours is a non-trivial complication since this generates several thresholds for different wage rates, corresponding to the mini-job earnings threshold with different working hours. The same strategic arguments as in the case of homogeneous hours will prevent firms from locating just below a mass point in the utility offer distribution. We demonstrate this in a version of the model with two hours sectors in Appendix C and note that the model quickly becomes intractable.

[11] This is true unless workers' marginal utility of leisure is zero, such that the utility function is independent of hours. We do not consider this trivial case.

strategies and profits differ across firms with different hours requirements. This implies that the wage distribution is not necessarily increasing as it would be with identical firms.

#### SSC and Taxes

The SSC rate, $t^{ssc}$, is constant for all earnings above the threshold and for workers whose employment constitutes a second job (see equation 1). The SSC rate is set to the average legal value over the analysis period (20.69 %). The income tax rate, $t^{inc}$, also depends on household characteristics and thus varies across individuals. These characteristics are not observed in our administrative data, so for our simulation as well as the empirical part, we impute tax rates based on an auxiliary survey data set and a micro-simulation model which accounts for the most important aspects of the German tax-transfer system (Appendix D). Tax rates of type-$s$ workers vary between roughly 8 % and 20 %. Approximately half of type-0 workers do not pay any income taxes even when they exceed the minijob threshold, since they do not exceed the general tax allowance. Tax rates of the remaining type-0 workers amount to up to 20 % (Appendix D).

#### Worker heterogeneity among type-0 workers

We have argued that taking into account individual tax rates is important since some individuals without another job (type-0) have higher taxes than others. On top of small tax notches, there may be other reasons why workers accept jobs above the threshold. For example, they may have very bad outside options, meaning that they accept job offers even if a job's earnings imply very low net earnings. To take these differences into account, we add worker heterogeneity within the population of type-0 workers: A fraction $\theta$ of type-0 workers only seek minijobs and do not accept any offers with earnings above the threshold. We designate these exclusive minijobbers type-0$m$. Their labor supply is thus conditional on obtaining the tax exemption. The remaining type-0 workers, called type-0$a$, accept all offered jobs if they have no small job - and will engage in on-the-job search moving to any job with higher utility in the market for low-paid employment. Their budget set exhibits a notch at the minijob threshold which varies across individuals. Both groups of type-0 workers thus generate an incentive for firms to offer earnings below or at the threshold.

#### Worker mobility and firm size

As in the case without variation in hours, workers either accept all jobs (type-$s$ and $0a$) or only minijobs with earnings up to the threshold (type-$0m$). We thus adapt equation 2 accordingly. As before, in equilibrium the flow of workers into and out of jobs with utility no greater than $v$ must coincide:

$$\lambda^j F(v) u^j = \begin{cases} [\delta^j + \lambda^j(1 - F(v))] G^j(v)(n^j - u^j) & \text{for } j \in (s, 0a) \\ [\delta + \lambda^0(1 - F(v|z \leq z^*))F(z^*)] G^{0m}(v)(n^{0m} - u^{0m}) = \lambda^0 F(v) u^{0m} & \text{for } j = 0m \ \& \ z \leq z^* \end{cases} \tag{7}$$

where $(1-F(v|z \leq z^*))F(z^*)$ describes the probability of another firm's offer exceeding $v$ with earnings at or below the threshold. For an individual firm with hours $h$, the strategic choice is then whether to offer utility $v(z, h)$ or utility $v(z', h)$, where strictly speaking the choice variable is the wage rate, but given fixed hours, different wage rates monotonously translate to different earnings and utilities. Profits are then given by

$$\pi(w) = \begin{cases} (p - w)h(\frac{n^s \kappa^s}{(1+\kappa^s(1-F(v(w))))^2} + \frac{(1-\theta)n^0 \kappa^0}{(1+\kappa^0(1-F(v(w))))^2} + \frac{\theta n^0 \kappa^0}{(1+\kappa^0(1-F(v(w)|z \leq z^*)))^2}) & \forall z \leq z^* \\ (p - w)h(\frac{n^s \kappa^s}{(1+\kappa^s(1-F(v(w))))^2} + \frac{(1-\theta)n^0 \kappa^0}{(1+\kappa^0(1-F(v(w))))^2}) & \forall z > z^* \end{cases} . \tag{8}$$

**Characterization of Equilibrium**

The model does not have a closed-form solution, but we can sketch some important characteristics of the equilibrium solution. First, variation in hours implies that the *utility offer distribution* will be smooth. The utility distribution will not exhibit a mass point or gap: If there was a mass point in the utility distribution, a firm could increase profits by slightly increasing the offered wage (and therefore utility). As hours of work are assumed to be distributed continuously, utility $v(z^*)$ varies across firms. We have a wage threshold function $w^*(h)$, where $w^*(h)\ h \equiv z^* \ \forall\ h$. The attractiveness of firms' offers will depend importantly on whether or not a given wage offer corresponds to an earnings level that lies above or below the earnings threshold $z^*$: This is because for some workers (the any jobbers, $0a$) utility decreases discontinuously at $z^*$ and a positive number of workers (the exclusive minijobbers, $0m$) do not accept any offers above. Given a smooth utility offer distribution this implies that there will be a gap in the distribution of earnings above $z^*$ because firm size drops discontinuously when earnings offers exceed the threshold. By contrast, there will be no gap below the threshold. The key difference to the case with homogeneous hours is that a mass point at $z^*$ does not imply that firms offering $z^* - \varepsilon$ will attract discontinuously less workers than those offering $z^*$ - attractiveness depends on the ranking of job offers, in terms of utility, not earnings.

Second, in equilibrium, earnings offers will be set such that firms with different hours requirements maximize their profits given other firms' earnings offers and hours requirements. The decision between offering earnings below, at or above the threshold will be assessed by firms according to their hours requirements: Firms with sufficiently small hours ($h \leq \bar{h} = \frac{z^*}{p}$) will offer earnings below the threshold. By contrast, firms with sufficiently high hours ($h > \bar{\bar{h}} = v^{-1}(w^*, \underline{v})$) will offer earnings above the threshold.[12] For firms with hours requirements between these two thresholds ($\bar{\bar{h}} > h > \bar{h}$), the trade-off described by Burdett and Mortensen (1998) holds: higher marginal profits per worker (the first term in equation 9 representing the first order condition of profit-maximization for a firm with hours requirement $h$) may be offset by a smaller firm size (the second term in equation 9). Identifying the best offer for these firms is not simple in our setting as firm size of a given firm is not monotonously increasing in the offered wage rate. Exceeding the threshold might even lead to a decrease in firm size as type-0 workers either suffer a utility drop at the threshold (any jobbers, type-0$a$) or do not accept any jobs with $z > z^*$ (exclusive minijobbers, type-0$m$).

$$\frac{\partial \pi(w)}{\partial w} = -h\left(\frac{n^s \kappa^s}{(1+\kappa^s(1-F(v(w))))^2} + \frac{(1-\theta)n^0\kappa^0}{(1+\kappa^0(1-F(v(w))))^2}\right) \tag{9}$$

$$+ (p-w)h\left(\frac{2n^s {\kappa^s}^2 \frac{\partial F(v(w))}{\partial v(w)} \frac{\partial v(w)}{\partial w}}{(1+\kappa^s(1-F(v(w))))^3} + \frac{2(1-\theta)n^0 {\kappa^0}^2 \frac{\partial F(v(w))}{\partial v(w)} \frac{\partial v(w)}{\partial w}}{(1+\kappa^0(1-F(v(w))))^3}\right) \tag{10}$$

The change in firm size by increasing the wage rate crucially depends on $\frac{\partial v(w)}{\partial w}$ which varies between firms with different hours. Assuming a Cobb Douglas utility function, $v(w)$ is a monotonously increasing concave function in all points except the threshold where $v(w)$ drops discontinuously.

Further, $v(w)$ varies between firms. There is an optimal number of hours $h^*$ in the sense that for any given wage $w$, utility is largest for the firm with $h = h^*$. The popularity of a firm depends on the preference parameter $\alpha$. At a given $w$, $\frac{\partial v(w)}{\partial w}$ is larger for 'better' firms. Profit-maximizing earnings of 'better' firms are thus more likely to be high and above the threshold. Firms with low hours requirements need to offer a high wage rate to reach threshold earnings. Since $v(w)$ is concave, a high $w$ implies a small value of $\frac{\partial v(w)}{\partial w}$. Profit-maximizing earnings are likely to be small and below the threshold. In

[12] This holds as long as the reservation utility level is strictly positive. As $v(.)$ is monotonously increasing in $w$ in all points except at the threshold $v(w^*, h) < \underline{v}$, this implies that a firm with $h$ or more will not attract any workers by offering $w \leq w^*$.

the end, whether a firm offers earnings above the threshold depends on the equilibrium offer distribution $F(v)$. $\frac{\partial F(v(w))}{\partial v(w)}$ needs to be sufficiently high for $w > w^*$ such that enough additional individuals who already hold a job (type-$s$) as well as individuals without a job who accept any job (type-$0a$) can be attracted to balance the loss in marginal profit *as well as* the loss of workers who only accept minijobs (type-$0m$ workers).

#### Simulation

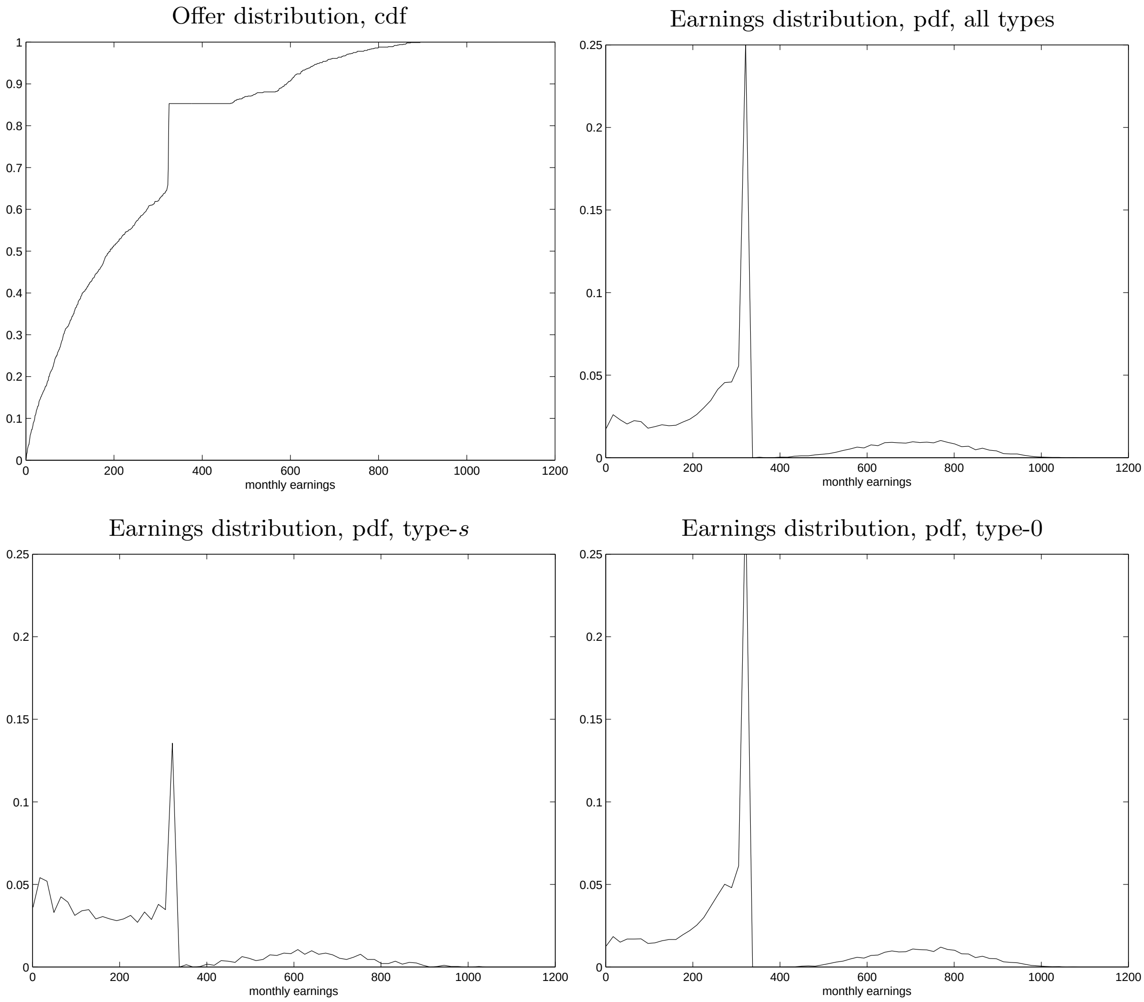
Since we have no analytic solution for the equilibrium earnings distribution, we solve for it as the solution to a fixed-point problem (in $F(z)$), see appendix F.2. Figure 4 presents results based on simulating the model using realistic parameter values (estimation is relegated to section 5). The simulated cumulative earnings offer distribution exhibits a continuously decreasing slope up to the threshold. The jump at $z^*$ implies a mass point in the density. It is flat above the threshold where the density exhibits a gap. From roughly 500 € it increases again up to approximately 850 € which is the highest earnings level in the simulated market. This translates into a realized earnings distribution which slowly increases up to a discontinuous mass point at $z^*$ where it sharply drops. This is consistent with the observed distribution (figure 1 above). The increase in the predicted distribution is driven by type-0 workers. The density for type-$s$ workers up to the threshold is slightly decreasing. This is not the case in our data although the observed density is indeed much flatter for type-$s$ workers (figure 1).

## 4 Data

We use administrative data collected by the German social security system. The Sample of Integrated Labor Market Biographies (SIAB) is a representative two percent sample of employees in the private sector.[13] For the present analysis, SIAB is the most appropriate data set. First, accurate total gross earnings for a period of an employment spell are observed. Second, with approximately 1.6 million sampled employees, the sample size is large enough to include a substantial number of individuals holding second jobs. It does not include civil servants and the self-employed. Third, SIAB includes complete employment biographies of the sampled individuals. We can, therefore, differentiate between first and second jobs and observe, for example, the exact day a second employment spell starts. It

[13] *Stichprobe der Integrierten Arbeitsmarktbiografien.* We use the weakly anonymized version of the data via on-site use at the research data center of the IAB in Berlin.

**Figure 4:** Offer and earnings distribution by types of workers




*Notes*: Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0 workers have or seek a low-paid job but have no other job. The minijob threshold is at 325€/month. Simulation is based on 100 firms and a roughly 13000 individuals. Underlying parameter values as estimated in section 6.1: $\lambda^s = 0.0329$, $\lambda^0 = 0.2506$, $\delta^s = 0.0570$, $\delta^0 = 0.0379$, $\theta = 0.3715$, $\alpha = 0.8648$, $p = 8$. Optimization error is assumed as discussed in section 5.2 ($\sigma = 71.21$).
*Source:* own calculations.

also includes unemployment spells. Fourth, tax-advantaged minijobs constitute a separate job category. We can therefore rule out that bunching of second job earnings at the minijob threshold is driven by misclassification. Fifth, we observe important individual and firm characteristics: age, sex, occupation and education, and for firms, industry sector, size and wage structure. The data have two limitations. First, the number of hours worked is not precisely measured. Hours information only exists in broad categories. Second, we do not observe information about earnings of spouses or other income sources. This information is however needed to calculate individual income tax rates. We thus use household data to complement our administrative data and impute individual tax rates. The German Socio

Economic Panel (GSOEP)[14] is a yearly representative household survey which includes information on labor market status, earnings, hours and all tax-relevant individual and household information.

## 4.1 Sample definition

We restrict the sample to employees[15] aged 17 to 65 years. For individuals with two employment spells, a second job is the one with lower earnings (very few individuals have more than two jobs, these are dropped). We do not classify very short overlaps between jobs (less than 5 days) as parallel employment spells. Interruptions of less than one month of otherwise similar spells are ignored unless the interruption is filled by another spell. We exclude employees who additionally receive benefits from unemployment insurance or social assistance. Due to high benefit withdrawal rates, low-paid employment is not attractive for these individuals. We include spells which ended after April 1999 or started before March 2002 in the analysis. Every spell is classified as type-$s$ or type-0, depending on whether individuals held a full-time job or not.

- Employed type-0 workers currently hold a single job with earnings of less than 800 €. Unemployed individuals who are observed to be job-seeking are classified as type-0 if they held a job paying up to 800 € at least once in the sample period and no full-time job.

- For type-$s$ workers, the small job they have or are seeking is a second job. Their first job is mostly a full-time job.[16] In absence of information on job-seeking for the whole sample, we restrict our sample to individuals who had at least one second job with earnings up to 800 € in the analysis period.

Distinguishing job-seekers from non-participants always includes a degree of arbitrariness, especially in absence of information on job search intensity. We therefore repeat the analysis with an alternative definition of who is job-seeking (see appendix I). Further, earnings of type-$s$ workers' full-time jobs are taken as given. Movements of individuals between type-0 and type-$s$ are very rare and we treat the initial spell of any such series as a censored spell.[17] The market of low-paid jobs is dominated workers who have no other

[14]See Wagner et al. (2007) for a detailed description.

[15]We exclude trainees, interns, employees in the military, individuals in old-age retirement, freelance home workers, people with special needs, short-term employees, seamen and artists.

[16]We use all employment spells with earnings above 1000 €. We choose this figure to be sensibly above the maximum value for the market of low-paid jobs, to avoid individuals switching between markets when earnings vary.

[17]For example, if a worker of type 0 is observed to start a full-time job, the first spell is censored.

job (type-0). Type-$s$ account for around 10 % of all employment spells in our sample (table 1). [18] While 84 % of type-0 workers have a small job, this is only the case for 38 % of type-$s$ workers. While a high fraction of job-seeking spells are right-censored (69.32 % for type-$s$ and 58.87 % for type-0, table 1), by far the most frequent reason for censoring is the end of the analysis period. The probability of being right-censored is considerably lower for employment spells (33.31 % for type-$s$ and 39.35 % for type-0). As minijobs are only included in the data as of April 1999, spells which started before April 1999 are left-censored. This is the case for roughly 20 % of the sample. Job separations (60.90 % for type-$s$ and 51.91 % for type-0) are more frequent than job-to-job transitions (5.8 % for type-$s$ and 8.73 % for type-0).

**Table 1:** Total number of spells by type and employment

| Status \ Type | s | 0 |
|---|---|---|
| Job-seeking | 49,984 | 55,592 |
| *job-seeker found small job* | 16,290 | 19,281 |
| *right-censored* | 33,694 | 36,311 |
| *left-censored* | 11,176 | 6,485 |
| Employed in small job | 30,121 | 287,255 |
| *small job to small job* | 1,690 | 23,803 |
| *small job separation* | 17,394 | 138,861 |
| *right-censored* | 11,037 | 124,591 |
| *left-censored* | 10,508 | 80,814 |

*Notes*: Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0 workers have or seek a low-paid job but have no other job. By far the most frequent reason for right-censoring is the end of the analysis period.

*Source:* SIAB; own calculations

## 4.2 Hours and taxes

While SSC rates are homogeneous in Germany, income tax rates depend on household characteristics. This implies that the size of the notch at the threshold varies between workers. In particular, marital status and spousal income are crucial. Since we have no information on these characteristics in our administrative data, we impute tax rates using

[18] The same proportion also holds for the number of individuals. For example, in June 2001, of all individuals holding a small job, for 96,789 individuals in our sample this was their first job, for 9,441 their second job.

detailed information on all tax-relevant characteristics provided by the GSOEP (Junge, 2017). For the sake of calculation speed we allocate individuals to three tax groups instead of allowing for continuous variation. Type-$s$ workers face larger tax rates than type-0 workers (table 2), and this is mainly driven by first job earnings. Appendix D presents the procedure for imputing tax rates.

The model outlined in the previous section assumes continuous variation in hours of work. Since we do not have information on hours of work in the data, we use the GSOEP to impute hours of work based on variables included in the SIAB: sex, gross earnings, sector and education. As the hours distribution resembles a log-normal distribution (see figure 3), a generalized linear model with log link is used for imputation. We then predict individual hours of work in our sample and let firms draw from the smoothed predicted empirical hours distribution. Mean predicted hours per week are approximately 9 for type-$s$ workers and 13 for type-0 workers (table 2). The average estimated hourly wage rate is approximately 5.50 € for both worker types.[19] We relegate details on imputation of hours to appendix E.

**Table 2:** Imputed hours, wage and tax rates

| | type-$s$ | | type-0 | |
|---|---|---|---|---|
| | mean | std | mean | std |
| weekly hours | 8.95 | 4.61 | 12.84 | 7.66 |
| wage rate | 5.56 | 1.48 | 5.76 | 1.37 |
| tax rate | 32.61 | 3.27 | 28.03 | 6.02 |

*Notes:* Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0 workers have or seek a low-paid job but have no other job. Tax rates include SSC (20.69 pp.).

*Source:* SIAB, GSOEP wave 2001, own calculations.

## 4.3 Descriptives

The earnings response to the tax exemption is very large. The observed earnings distribution of low-paid jobs as first jobs features a large peak at the threshold for minijobs (figure 1 in section 2, left panel). This group of workers (type-0) is subject to the minijob threshold implying a strong incentive to locate below the 325 € threshold. Note that large bunching at a notch in the budget set does not necessarily imply a high labour supply elasticity as

[19]Note that this may appear low given the minimum wage of 8.50 € that was introduced a decade later. However, it corresponds to other information from our GSOEP household sample and the model fit is fairly good (see appendix E).

in the case for kinks (Kleven and Waseem, 2013). In a certain earnings interval, reducing earnings to the minijob threshold is the dominant strategy independent of preferences (if leisure and consumption are normal goods). For second job earnings there is no incentive to locate at the 325 € threshold as the minijob regulations do not apply. Nevertheless, the earnings distribution of type-$s$ workers features a clear peak at the minijob threshold and hardly any mass beyond (right panel of figure 1), suggesting important equilibrium effects.[20] Minijobs constitute a separate job category in our data set. As this information is relevant for the administrative process, there is virtually no measurement error with respect to classifying employees in small jobs into type-$s$ and type-0 workers. Firms seem to offer contracts based on job-seekers of minijobs having no full-time employment, consistent with our assumption of both types drawing from the same take-it-or-leave-it offer distribution. Furthermore, the earnings densities of both types increase gradually up to the mass point which is of similar relative size. The fraction of employees below the threshold is higher for type-$s$ ($\approx$ 90 %) than for type-0 workers ($\approx$ 75 %). We therefore allow the offer arrival rate to vary between types in our model. This is consistent with observed differences in job-seeking spells, which are shorter for type-0 workers (see table 3). The average duration in a small job is similar for both types.

**Table 3:** Spell durations in months

| Status \ Type | s | f |
|---|---|---|
| Duration job-seeking | | |
| *uncensored* | 5.76 | 4.14 |
| *only right-censored* | 9.40 | 4.62 |
| *only left-censored* | 12.74 | 9.42 |
| *left+right-censored* | 10.87 | 6.2 |
| Duration of job spell | | |
| *uncensored* | 5.56 | 5.45 |
| *only right-censored* | 10.75 | 10.85 |
| *only left-censored* | 10.32 | 12.19 |
| *left+right-censored* | 28.94 | 28.09 |
| Duration in employment (across 1+ jobs) | | |
| *uncensored* | 6.10 | 8.49 |
| *only right-censored* | 11.76 | 14.77 |
| *only left-censored* | 11.94 | 17.72 |
| *left+right-censored* | 29.37 | 29.04 |

*Notes:* Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0 workers have or seek a low-paid job but have no other job.
*Source:* SIAB, own calculations

[20] Interestingly, the small spike slightly below 200 € is also an indication of a population labor supply effect. Workers receiving unemployment benefits have an incentive to earn at most 165 €. Although our sample excludes these workers, we see a small spike at 165 €.

The distribution of sectors are fairly similar for small jobs across types (see table 4, columns 1 & 2): The dominant sectors are "cleaning, security and other firm services" as well as wholesale/retail, with the former more frequent as a second job and retail more common as a first job. This also supports the assumption that both types draw offers from the same job offer distribution. By contrast, in line with our assumptions, the (mainly) full-time jobs of second-job holders are different: Here manufacturing is the main sector (third column in table 4). This suggests that low-paid employment is indeed a separate market.

**Table 4:** Distribution of sectors, 2000-2002

| Sector | Low-paid job type-0 | type-$s$ | 1st job of type-$s$ |
|---|---|---|---|
| D Manufacturing | 11.67 | 10.27 | 30.02 |
| G Wholesale & retail | 22.22 | 13.74 | 13.53 |
| H Hotels & restaurants | 9.08 | 8.62 | 2.21 |
| I Transport | 4.77 | 8.70 | 5.87 |
| K Cleaning, security & other firm services | 18.98 | 28.70 | 10.52 |
| L Public admin | 2.18 | 2.97 | 7.62 |
| N Health | 10.87 | 8.08 | 12.72 |
| O Other services | 7.74 | 9.18 | 4.21 |

*Notes:* Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0 workers have or seek a low-paid job but have no other job.
*Source: SIAB*

In our sample many workers in small jobs are relatively young women (table 5). East Germans are slightly over-represented and over 90% have at most vocational training. Job-seekers of type 0 are relatively more likely male, East German and not highly educated. Survey data shows that type-0 workers are predominantly housewives/-men, students, pensioners and registered job-seekers, with housewives constituting the largest and registered job-seekers the smallest group (Körner et al., 2013). Only roughly one quarter say that they do not work more because they cannot find a suitable position, in line with our differentiation between workers willing to gain higher earnings and workers exclusively seeking minijobs ($0a$ and $0m$). Furthermore, minijobs rarely serve as stepping stones to better-paid jobs (Freier and Steiner, 2007), such that we can treat this market as independent. The gender ratio of workers holding or seeking side jobs is fairly balanced (table 5). The share of East German workers is smaller and type-$s$ workers are slightly better educated than type-0 workers. The age distribution is roughly similar across types.

**Table 5:** Socio-demographic characteristics by type, 2001

| | type-$s$ | | type-0 | |
|---|---|---|---|---|
| | job-seeking | employed | job-seeking | employed |
| Female | 48.20 | 45.15 | 52.67 | 76.26 |
| Age | 36.51 | 39.73 | 40.06 | 38.77 |
| East German | 9.07 | 7.76 | 29.82 | 13.22 |
| *Education* | | | | |
| Intermediate | 12.99 | 12.97 | 28.13 | 24.24 |
| Voc. training | 74.81 | 74.71 | 65.16 | 67.16 |
| Grammar | 6.72 | 6.15 | 3.71 | 4.84 |
| University | 5.48 | 6.17 | 3.00 | 3.76 |
| *Characteristics of first job ("full-time job") of type-$s$ employees* | | | | |
| Monthly earnings (euro) | 2,025 | 2,131 | . | . |
| Part-time | 17.4 % | 18.3 % | . | . |

*Notes:* Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0 workers have or seek a low-paid job but have no other job.
*Source:* SIAB; own calculations

# 5 Identification and Estimation

We seek to estimate eight parameters:[21] Two offer arrival rates ($\lambda^s$ and $\lambda^0$), two job destruction rates ($\delta^s$ and $\delta^0$), the relative weight of consumption and leisure in the utility function ($\alpha$), the fraction of type-0 workers who do not accept jobs with earnings exceeding the threshold ($\theta$), the standard deviation of optimization error and the level of productivity per hour ($p$). In this section we first discuss identification of these parameters (section 5.1) before presenting our two-step maximum likelihood (ML) procedure which is used for estimation (section 5.2). Note that there is no single parameter summarizing the equilibrium effects of the minijob policy. Equilibrium effects arise whenever firms set wage offers according to individual worker preferences. Parameters quantifying labor market frictions, workers' preferences and population sizes of different worker types all play a role in determining the size of equilibrium effects. To highlight spillover effects of other workers' preferences, we perform counterfactual simulations. We compare removing the MWP tax exemption or smoothing the subsidy rate to reduce the incentive for firms to post offers at the specific minijob threshold (6.3). In an appendix, we focus specifically on the spillover effects across worker types by limiting the market to one worker type (appendix H). Similarly, job creation elasticities[22] could be calculated numerically by simulations changing the tax rate. We refrain from doing this due to space considerations.

[21] We observe individuals accepting jobs with very low utility. This suggests that the level of reservation utility is very low. We set it to zero.

[22] Labor supply and demand elasticities cannot be calculated separately in our framework.

### 5.1 Identification

While our arguments for identification are largely constructive, we use maximum likelihood to estimate the model (see below). The frictional parameters are identified by observed durations in employment or job-seeking. Type-$s$ workers are assumed to accept all jobs. Their job offer arrival rate $\lambda^s$ is thus identified by the observed duration of job-seeking for type-$s$ workers. An unobserved fraction of type-0 workers also accepts all offers if they have no small job - we denote these by $0a$. Type-0 workers who accept jobs above the threshold are necessarily of type $0a$, who accept all jobs. The duration of job-seeking of this observed group identifies the job offer arrival rate of type-0 workers, $\lambda^0$. Duration in employment (for different types, and including job-to-job mobility) identifies $\delta^s$ and $\delta^0$. The upper support of the wage distribution identifies the underlying productivity of jobs. Alternately, the job offer distribution is non-parametrically identified from the distribution of accepted earnings by formerly job-seeking type-$s$ workers who accept all jobs.

The distribution of realized earnings (in particular, the proportion of earnings below, at or above the threshold) is informative about both the preference parameter $\alpha$ and $\theta$, the proportion of individuals who exclusively accept minijobs. A high $\theta$ implies a high fraction of workers who do not accept offers with earnings $z > z^*$, increasing the incentive for firms to offer earnings at or below the threshold. The same is true for a low $\alpha$ implying a low relative weight of earnings (vis-à-vis leisure) in the utility function. Differences in durations job-seeking across worker types are also informative of $\theta$. For job-seekers who only accept minijobs (type-$0m$ workers) the probability of a match in any one period is $\lambda^0 F(z^*)$. For other workers this probability is simply $\lambda^0$ or $\lambda^s$, respectively. Conditional on $\lambda^0$ and the offer distribution, the difference in job-seeking duration between type-$s$ and 0 is thus informative about $\theta$. The same is true for the difference in job-seeking duration between type-0 workers who accept earnings offers below and above the minijob threshold.

### 5.2 Estimation procedure

We use a two-step ML procedure. In the **first step**, the frictional parameters are estimated based on duration data. The preference parameters are then estimated in the **second step** based on data on labor market spells and earnings. Although less efficient, joint estimation is numerically extremely costly in our case.

For the **first step**, we use workers' relevant employment spells. The duration in employment is used to estimate the job separation rates following equation 11, which consists of the probability that an employment spell is drawn and the exponentially distributed duration of employment, $t_{emp}$, with transition rate $\delta$,

$$L_{e1}^{j}(t_{emp};j) = \left(1 - \frac{\delta^j}{\delta^j - \lambda^j}\right) (\delta^j)^{1-d} exp[-\delta^j(t_{emp})] \quad \text{for } j \in (s, 0), \tag{11}$$

where $d$ is a dummy for censored observations (left- or right-censored). We estimate job offer arrival rates using the duration job-seeking (equation 12 where t is spell length). Exclusive minijobbers (type-$0m$ workers), of course, do not accept jobs with $z > z^*$. Their job-seeking duration depends on the offer distribution. We therefore only use type-0 workers who accept earnings above the threshold.

$$L_{u1}^{j}(t;j) = \frac{\delta^j}{\delta^j - \lambda^j} (\lambda^j)^{1-d} exp[-\lambda^j(t)] \quad \text{for } j \in (s, fa) \tag{12}$$

The **second step** uses information on durations as well as earnings in order to estimate $\theta$ and $\alpha$. While the endogenous job offer distribution $F(.)$ depends on structural parameters, we do not have an analytical expression. In our setting the restrictions on the earnings distribution help identify $\alpha$, the parameter of the utility function and $\theta$, the unobserved fraction of type-0 workers who do not accept any jobs with $z > z^*$ (type-$0m$). In contrast to the non-parametric two-step procedure proposed by Bontemps et al. (1999)[23] we therefore calculate $f(.)$ and $F(.)$ by numerically solving the firms' problem of choosing the profit maximizing earnings level in every iteration (see Appendix F for more details).

The likelihood contribution for a job-seeker (currently without a small job) is composed of the probability of having a small job, the spell duration and the realized wage of the following employment spell (if observed, i.e. $d_r = 0$).

$$L_{u2}^{j}(t,z;j) = \frac{\hat{\delta}^j}{\hat{\delta}^j - \hat{\lambda}^j} (\hat{D}_u^j)^{1-d} exp[-\hat{D}_u^j t] * \left(\int_{-\infty}^{\infty} \hat{f}_z(z+\eta) dh(\eta)\right)^{(1-d_r)} \quad \text{for } j \in (s, 0a, 0m) \tag{13}$$

[23]While Bontemps et al. (1999) similarly have no analytical expression for the job offer distribution, they can apply the following two-step procedure: First non-parametrically estimate the cdf and pdf of the realized wage distribution ($G$ and $g$). Using these estimates and their relation to the offer distribution (equation 7), the likelihood contributions can be expressed in terms of the realized wage distribution and the model parameters. Once the transition parameters are estimated from workers' mobility patterns, these are used to transform the observed distribution $G(.)$ to the offer distribution $F(.)$ which is the object required in the likelihood. This strategy only exploits equations with respect to *workers'* behavior, the offer distribution is basically treated as exogenous. Firms' behavior leading to the endogenous offer distribution is not exploited, but is key to our identification argument.

where $\hat{D_u^j}$ denotes the estimated arrival rate of acceptable offers. For $j \in (s, 0a)$ it holds that $\hat{D_u^j} = \hat{\lambda}_j$. For $j = 0m$, $\hat{D_u^j} = \hat{\lambda}^j \hat{F}_z(z^*)$. The third part of equation 13 is the probability that the accepted job has true earnings $\tilde{z_u} = z_u + \eta$ where $z$ denotes observed earnings and $\eta \sim N(0, \sigma^2)$ represents random optimization error. Firms might not always offer exactly desired earnings because, for example, weekly hours or the wage rate often are round numbers. Frictions are a standard assumption in the bunching literature where bunching is usually found as a hump around the threshold rather than an exact spike (Saez, 2010).[24]

For likelihood contribution of a small-job-holder is then

$$\begin{aligned} L_{e2}^j(t,h,w;j) = &\left(1 - \frac{\hat{\delta^j}}{\hat{\delta^j} - \hat{\lambda^j}}\right)(\hat{D_e^j}(h,w))^{1-d} exp[-\hat{D_e^j}(h,w)t] * \\ &(\delta^{1-J2J}\hat{\lambda^j}(\hat{D_e^j}(h,w))^{J2J})^{(1-d_r)} * \\ &\left(\int_{-\infty}^{\infty} \hat{g_z}(z_e+\eta)dh(\eta)\right)^{(1-d_r)} \quad \text{for } j \in (s,0a,0m), \end{aligned} \tag{14}$$

where $J2J$ indicates whether an uncensored employment spell ends with a job-to-job transition ($J2J = 1$) or a separation ($J2J = 0$). It holds that $\hat{D_e^j}(v) = \hat{\delta^j} + \hat{\lambda^j}(1 - \hat{F}(v))$ for $j \in (s, 0a)$. For $j = 0m$, $\hat{D_e^j}(v) = \hat{\delta^j} + \hat{\lambda^j}(1 - \hat{F}(v|z \leq z^*))\hat{F}(z^*)$. The third part is the probability of a separation ($\delta$) or a transition to a better small job ($\hat{D_e^j}(h_e, w_e)$), if observed (i.e. if $d_r = 0$). The last part is the probability that the current job has true earnings $\tilde{z_e} = z_e + \eta$. The density of realized earnings, $g_z$, is derived from equation 7 and the estimation of the offer distribution. While we observe whether an individual is of type $s$ or 0, we cannot distinguish whether or not a worker would be willing to accept an offer above the threshold (whether she is type-$0a$ or $0m$) if observed wages are below the threshold or not observed. For $k \in (u, e)$ the likelihood for type-0 workers is therefore

$$L_k^f(t,z) = \begin{cases} \theta L_{k2}^{0m}(t,z) + (1-\theta)L_{k2}^{0a}(t,z) & \text{for } z \leq z^* \\ L_{k2}^{0a}(t,z) & \text{for } z > z^* \end{cases} \tag{15}$$

We then numerically estimate $\alpha$, $\theta$ and $\sigma$ by maximizing the likelihood contributions (see appendix F for further details).

[24]However, we do not allow for the case that true earnings are below the threshold while observed earnings are above or vice versa: When $z > z^* > \tilde{z}$ or $z < z^* < \tilde{z}$ we set $z = \tilde{z}$. Intuitively, the observed distribution of earnings drops very sharply at exactly the threshold while it decreases much more gradual to the left (figure 1). We argue that the cost of crossing the threshold is so high that it exceeds potential adjustment costs or costs for exact declaration.

# 6 Results

This section presents estimation results, model fit and simulates counter-factual policy reforms. The equilibrium effects of the tax system for other groups in the labor market is highlighted by simulating the removal of the minijob MWP policy.

## 6.1 Estimation results

The rate at which workers are separated from their small jobs is larger for workers who have a full-time job (type-$s$) than for workers who only have a small job (type-0, table 6). Small jobs as second jobs are terminated every 18 months, small jobs for individuals without full-time job last on average 26 months. Type-$s$ workers receive an offer on average every 30 months while type-0 workers receive three job offers per year. Second jobs are, thus, less stable and workers need more time to find one (this makes sense as workers seeking second jobs may have less time to search for a job). We estimate that 37 % of workers without a full-time job (type-0 workers) in the market do not accept offers with earnings exceeding the threshold. This group of workers represents roughly two million individuals who are only in the market due to the tax exemption. At first sight the minijob policy thus seems to achieve its main objective to facilitate new employment relationships. Conversely, this also implies that more than 60 % of workers whose main job is low-paid employment would also work if there were no subsidy (type-0$a$ workers). From the government's point of view the increase in employment thus comes at the expense of lower taxes and SSC revenues from this group. Intensive margin effects of type-0$a$ workers are discussed in the following section when we simulate removing the tax exemption.

The elasticity of consumption (leisure) with respect to utility is 0.86 (0.14) and not statistically different from unity. The standard deviation of the optimization error is estimated to be 71 €/month. This implies, for example, that about 15 % of jobs with observed earnings of less than 250 € actually have true earnings at the threshold. The transition parameters are estimated very precisely. Standard errors are larger for the remaining parameters (table 6). The reason is that standard errors for the latter include the uncertainty rooted in the random elements of the estimation procedure (section 5).[25] We use the 95th quantile of the observed wage distribution as an estimator for the homogeneous

[25]Precision can be increased by basing these random procedures on more repetitions. The calculation of the offer distribution is, for example, repeated ten times per parameter set. Increasing this number increases precision but is costly in terms of computing time, which is limited at the on-site use of the data provider.

productivity $p$ which is 8 €/hour.[26]

**Table 6:** Parameter estimates

| Parameter | p.e. | s.e. |
|---|---|---|
| $\delta^s$ | 0.0570 | 0.0005 |
| $\delta^0$ | 0.0379 | 0.0002 |
| $\lambda^s$ | 0.0329 | 0.0002 |
| $\lambda^0$ | 0.2506 | 0.0028 |
| $\theta$ | 0.3715 | 0.0775 |
| $\alpha$ | 0.8648 | 0.1121 |
| $\sigma$ | 71.212 | 6.963 |
| $p$ | 8 | - |

*Notes:* p.e. – mean point estimate of 50 bootstrap repetitions, s.e. – standard deviation of 50 bootstrap repetitions, $\lambda^j$ – arrival rate for worker type $j$, $\delta^j$ – job destruction rate of worker type $j$, $\alpha$ – relative weight of consumption and leisure in the utility function, $\theta$ – fraction of type-0 workers who do not accept jobs with earnings exceeding the threshold, $\sigma$ – standard deviation of optimization error, $p$ – level of productivity per hour
*Source:* SIAB; own calculations.

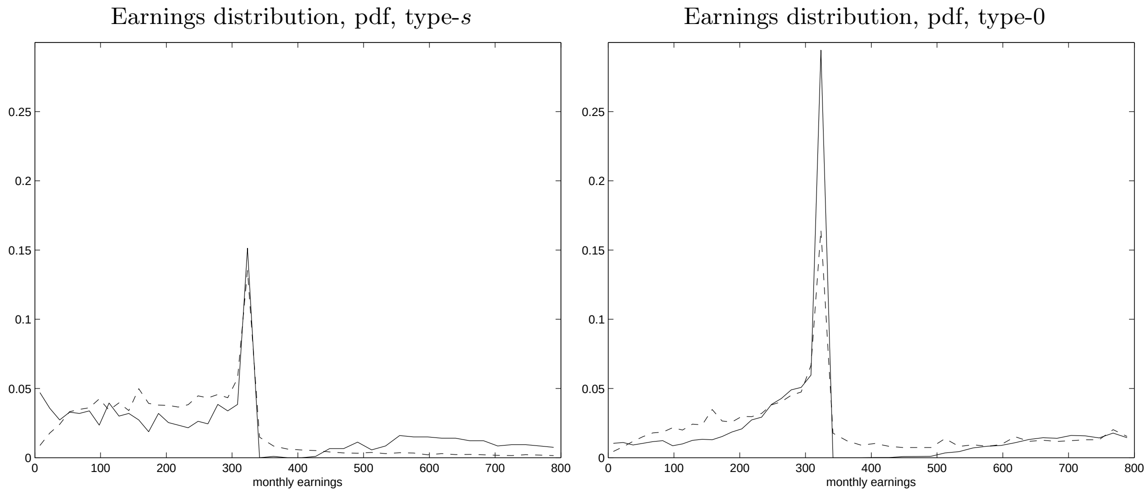
## 6.2 Model Fit

We simulate the low-paid market based on the estimated parameter estimates. The predicted realized earnings distribution reflects the main characteristics of its observed counterpart fairly well (figure 5). For type-0 workers it increases gradually up to a discontinuous mass point where it drops sharply. The mass beyond the threshold is relatively small. This is perfectly consistent with the observed distribution. The size of the mass point is over-estimated, though. For type-$s$ workers, by contrast, the size of the mass point in the predicted and observed distribution almost coincide. Our model also correctly predicts that the side job earnings distribution is fairly flat to the left of the threshold. By contrast, our model does not predict any mass slightly above the threshold although some workers are observed to be located there. As utility and profits could be increased by decreasing earnings to the threshold, our model cannot rationalize this.[27] A second limit concerns the highest earnings offered, which exceeds 1000 € per month in the simulation (this explains the jumps in density at the upper limit in figure 5) while we cut the observed earnings data at 800. Driving the upper support of the earnings distribution is our assumption on productivity which may be too high for some firms.

[26] Estimating $p$ jointly with the other parameters would increase the state space by a further parameter. This is costly in terms of computing time, which is limited at the on-site use of the data provider (see section F).

[27] Other studies interpret observed mass in a dominated region as evidence for "optimization frictions" (Kleven and Waseem, 2013). Although we indeed include an optimization error in the model, we do not allow earnings to cross the threshold as a result, since this would smooth the discontinuity at the threshold (section 5.2).

**Figure 5:** Offer and earnings distribution by types of workers




*Notes:* Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0 workers have or seek a low-paid job but have no other job. The minijob threshold is at 325€/month. The solid line represents the predicted distribution, the dashed line the observed one.
*Source:* own calculations.

The employment probability is fit very well for both worker types (table 7). The model also correctly predicts that hours, wages and earnings are higher for type-0 than for type-$s$ workers. The fit of the earnings level is relatively good albeit on average slightly high.

**Table 7:** Fit of moments

| | type-$s$ Mean | | type-0 Mean | |
|---|---|---|---|---|
| | obs | pred | obs | pred |
| $P(e)$ | 37.60 | 37.36 | 83.79 | 86.11 |
| $z$ | 223.62 | 279.91 | 320.16 | 365.97 |
| $w$ | 5.56 | 4.25 | 5.76 | 4.54 |
| $h$ | 8.95 | 13.94 | 12.84 | 18.41 |

*Notes:* Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0 workers have or seek a low-paid job but have no other job; $\bar{w}$ =mean wage, $\bar{h}$ =mean weekly hours, $\bar{z}$ =mean earnings, $P(e)$ =employment probability, *unrest.* =all earnings, *unrest.* $= z \leq 800$
*Source:* SIAB; own calculations.

## 6.3 Counterfactual Simulations: Remove or smooth tax exemption

The main objective of MWP policies is to create employment. This often comes at the cost of an incentive for already employed individuals to reduce hours of work (Bargain et al., 2010): The withdrawal of the subsidy usually implies high implicit marginal tax rates. Crossing the minijob threshold in earnings implies a substantial increase in the average tax rate, in Germany, i.e. there is a notch. This generates not only large bunching, but also incentives for many firms to offer minijobs, creating externalities for workers who are not directly affected by the tax exemption. We now evaluate these aspects using three policy reform scenarios which remove the discontinuity induced by the minijob tax exemption. First, we simulate completely removing the tax exemption. Second, we replace the minijob regulation by a smoother subsidy schedule which retains the objective to generate employment. It emphasizes the particular distortion induced by discontinuous tax policies like the German minijob regulation. Third, we predict the effects of a reform implemented in 2003 aimed at reducing the discontinuity at the minijob threshold.

For these policy simulations we need to consider labour demand. When tax exemptions are reduced, all exclusive minijobbers (type-$0m$ workers) leave the market. How many of these jobs are offered to other workers? I.e. how much substitution is there across workers? If jobs are lost because the productivity of the activity is too small to allow for an acceptable wage rate, substitutability is arguably low and the jobs are lost. To test the sensitivity of our analysis to labor demand we make two very different sets of assumptions. In the first scenario, we assume that our frictional parameters are exogenous and constant. The number of vacancies in the economy falls in response to the increase in labor costs and the reduction in labor supply - resulting in a constant job-finding rate. In a second scenario (relegated to appendix G), we assume that the offer arrival rate increases for all remaining workers such that the total number of offers per month stays constant.[28]

#### Removing the minijob tax exemption

When the tax exemption is removed, by assumption all exclusive minijobbers (type-$0m$) leave the market.[29] This amounts to roughly one and a half million employment relationships (table 8). The removal of the tax exemption also affects other workers as a result of firm responses.

As expected, the earnings offer distribution becomes smooth when the tax exemption is removed (left panel of figure 6). Since jobs with earnings above the threshold become

[28] Alternative scenarios are possible, e.g. assuming fixed and exogenous hours for given firms. The relative size of firms with different hours might then respond to a change in financial incentives. When tax rates increase, for example, leisure becomes relatively more attractive and so firms with fewer hours expand.

[29] Note that while this assumption is restrictive, our model includes both wage and employment margins - and the fraction of $0m$ workers is itself estimated. This contrasts to two frequently used methods to analyze tax exemptions such as the minijob rule: First, often the total number of minijobs is taken as the extent of the job creation due to the exemption. This assumes that *all* minijobs would be destroyed. Second, bunching estimators assume that tax exemptions only affect earnings and not participation (Tazhitdinova, 2017).

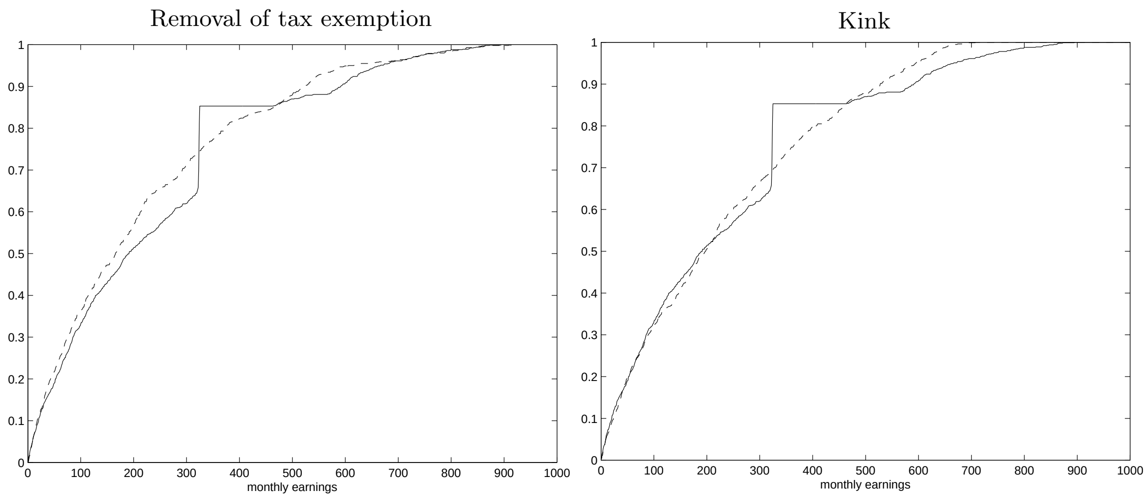
relatively more attractive for workers, firms offer more of these jobs. This reveals that firms are responsible for negative effects of the minijob regulation on earnings of already employed - average gross earnings of workers employed in the low-paid market increase (table 8) as lower-paid exclusive minijobbers ($0m$ workers) leave the market. However, the likelihood of offers *below* the threshold increases as well. The mass point at the threshold in the status-quo scenario is thus distributed over almost the entire support when the minijob tax exemption is removed. Why are there more low-earnings-offers in the market now? Our results show that workers who additionally hold a full-time job (type-$s$ workers) receive fewer offers of small jobs than type-$0m$ workers (see table 6). Since the latter have now left the market, the new equilibrium is less competitive. This means that the likelihood that another firm outbids a job offer decreases. Firms therefore offer less valuable jobs.

Equilibrium effects thus go beyond an earnings reduction: Whilst the population of exclusive minijobbers ($0m$ workers) creates a particular incentive for firms to offer minijobs rather than higher-paying jobs, the increased competition of workers also creates an incentive for firms to create good offers up to the minijob threshold. As a combined result of these two forces, the resulting realized earnings distribution of workers who additionally have a full-time job (type-$s$ workers) monotonously decreases as an increase in hours is overcompensated by a decrease in the wage rate. The average utility of type-$s$ workers also decreases. This implies that type-$s$ workers benefit more from the positive externality of the higher offer arrival rate than they lose due to the discontinuous tax schedule. The earnings distribution of small jobs for workers who do not additionally have a full-time job (type-0 workers), by contrast, increases up to approximately 500 € and decreases beyond. This difference mirrors the much higher offer arrival rate of type-0 workers (and potentially also lower income tax rates), driven by both higher wage rates and an increased number of hours of work. These workers are hampered by the minijob threshold in the sense that gross earnings are lower than they would be. But note that they still profit substantially from the tax exemption.

Why are workers with and without full-time jobs (type-$s$ and 0 workers) affected so differently in the market for small jobs? Firms offer considerably more jobs with earnings below the threshold when the tax exemption is removed resulting in lower average starting earnings (and wages). The opportunities of advancement also increase, though, as firms offer more jobs with earnings above the threshold as well. The latter is mainly beneficial for workers with no full-time job who accept offers above the threshold (type-$0a$ workers), as they receive considerably more offers.

Total hours and gross earnings decrease by roughly 23 % and 16 %, respectively (table 8). Tax revenues increase by 110 million Euro. This opens scope for compensation. When the additional tax revenue is equally distributed to all individuals by tax-free lump-sum transfers, net earnings and utility of all workers remaining in the market (types $s$ and $0a$) would increase strongly (table 8). Both worker types thus clearly benefit from the removal of the tax exemption. The reform is not Pareto improving in terms of utility, though, as the exclusive minijobbers (type-$0m$ workers) lose albeit being compensated. The tax exemption has redistributive effects: workers who would not otherwise work (with rich spouses, students and pensioners) gain at the expense of workers seeking low-paid employment independent of any tax exemption.

**Figure 6:** Cumulative earnings offer distribution - status quo vs. counter-factual




*Notes:* Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0 workers have or seek a low-paid job but have no other job. The minijob threshold is at 325€/month where the solid lines feature a jump. The solid line represents the status quo, the dashed line the counterfactual. The left panel analyzes the entire removal of the tax exemption; the right panel the replacement of the notch by a kink.
*Source:* SIAB, own calculations.

### Smoothing the tax schedule

In the following we simulate a reform which replaces the minijob regulations by a smooth subsidy schedule. Average tax rates increase gradually in the interval between 325 € and 800 € up to their full values. The complete tax exemption below 325 € remains. The notch is thus replaced by a kink. The objective is to prevent additional

**Table 8:** Effects of smoothing the discontinuity

| | Removal of tax exemption | | | | Kink | | | |
|---|---|---|---|---|---|---|---|---|
| Change in | Total | type-$s$ | type-0$a$ | type-0$m$ | Total | type-$s$ | type-0$a$ | type-0$m$ |
| $\bar{w}$ | 0.23 | -0.20 | 0.34 | . | 0.08 | 0.08 | 0.16 | -0.04 |
| $\bar{h}$ | 11.92 | 0.96 | 2.13 | . | 7.57 | 1.02 | 0.91 | 21.49 |
| $\sum h$ (%) | -22.58 | 0.44 | 1.67 | -100 | 10.04 | 1.69 | 0.98 | 38.87 |
| $\bar{z}$ | 91.70 | -4.56 | 52.81 | . | 37.97 | 5.30 | 8.99 | 99.97 |
| $\sum z$ (%) | -15.62 | -2.74 | 11.65 | -100 | 10.94 | 1.89 | 2.10 | 40.03 |
| $\bar{c}$ | 18.79 | -2.66 | -7.55 | . | 41.54 | 3.50 | 28.43 | 77.23 |
| $\bar{v}$ | 6.34 | -3.72 | -6.47 | -199.34 | 28.45 | 3.58 | 20.67 | 36.08 |
| jobs | -1644 | 0 | 0 | -1644 | 13.5 | 0 | 0 | 13.5 |
| taxes | 110.56 | -2.39 | 199.17 | -86.21 | 23.13 | 1.54 | -50.08 | 71.67 |
| *After lump-sum transfers* | | | | | | | | |
| $\bar{c}$ | 35.51 | 14.06 | 9.16 | . | 45.04 | 7.00 | 31.92 | 80.73 |
| $\bar{v}$ | 53.00 | 40.02 | 41.48 | -174.02 | 71 | 36.56 | 64.64 | 78.32 |

*Notes:* Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0$a$ workers have or seek a low-paid job but have no other job and would work even when fully taxed. Type-0$m$ workers have or seek a low-paid job but have no other job and would only work when not taxed; $\bar{j}$ – average of variable $j$, $w$ – hourly wage rate, $z$ – monthly gross earnings, $c$ – monthly net earnings, $h$ – weekly hours of work; $\bar{w}$, $\bar{z}$ and $\bar{c}$ in €; $\sum h$ and $\sum z$ in %; jobs in 1000s; taxes in million €; jobs and taxes extrapolated to population; $\bar{w}$, $\bar{h}$, $\bar{z}$, and $\bar{c}$ conditional on employment, $\bar{v}$ based on all individuals
*Source:* SIAB; own calculations.

distortions induced by a discontinuous tax schedule while positive employment effects are retained. Note that the withdrawal of the subsidy implies high implicit marginal tax rates in the interval between 325 and 800 €.

This hypothetical reform makes the tax schedule more generous, raising the question of whether additional workers (akin to type-0$m$) participate in the market. We make two assumptions regarding labor supply. We assume, first, that no additional individuals enter the labor market due to tax subsidies beyond 325 €. Second, we postulate that labor supply of exclusive minijobbers (type-0$m$ workers) varies with the average tax rate. In the status quo, type-0$m$ workers do not accept any jobs with earnings above the threshold. Now, the average tax rate increases gradually with earnings beyond the threshold. Type-0$m$ workers will thus also accept some jobs above the threshold. We assume that $\theta(1 - H(t(z)))n_f$ workers accept a job offer with earnings $z$, where $H(.)$ is the cumulative uniform distribution (equation 16). Intuitively, this implies that type-0$m$ workers gradually leave the labor market when earnings – and therefore average taxes – increase.

$$H(t(z)) = \begin{cases} 0 \quad \text{for } z \in (0, 325] \\ \frac{t(z)-325}{800-325} \quad \text{for } z \in (325, 800] \\ 1 \quad \text{for } z > 800 \end{cases} \tag{16}$$

The resulting earnings offer distribution is smooth (right panel of figure 6). Most firms which offered jobs at the threshold in the status quo now offer jobs with higher earnings. The profile of the offer distribution is now steeper above the threshold with the highest earnings offer being considerably lower than in the status quo. The reason is twofold. First, the high implicit marginal tax rate induced by the withdrawal of the subsidy creates an incentive for workers to reduce earnings. Second, a marginal increase in offered earnings now results in a marginal decrease of exclusive minijobbers (type-0$m$ workers) above the threshold also.

Relative to the status quo all types of workers increase their average gross earnings (table 8, right panel). This is driven by both wage rate and hours effects. Gross earnings of type-0$m$ workers increase the most, the average increase (9 €) for type-0$a$ workers is fairly modest. This is particularly true when compared to the increase resulting from the removal of the tax exemption (52.81 €). This again highlights the negative impact of the high implicit marginal tax rate introduced by the reform which has often been found in the previous literature on MWP policies (Bargain et al., 2010).

In terms of utility, all workers without full-time jobs (type-0 workers) - both those who only hold a small job due to tax subsidies and those who would hold one anyway - benefit substantially from the reform. Besides the modest increase in gross earnings, workers who accept jobs above the threshold (type-0$a$ workers) mainly benefit from the additional tax subsidies beyond 325 € leading to substantially higher net earnings. Exclusive minijobbers (type-0$m$ workers) benefit from a slight increase in employment because the likelihood of an acceptable offer increases for this type. As workers with an additional full-time job (type-$s$ workers) are not eligible, their increase in utility is much smaller but still positive. The smoothing of the discontinuity is thus also beneficial for workers not directly affected. Overall hours and gross earnings increase by approximately ten percent. Total tax revenue increases by 23 million €. From the government's point of view, the positive behavioral responses more than compensate for the increase in generosity of the tax schedule. A budget neutral version of the reform redistributes the additional tax revenues evenly to all individuals as lump-sum transfers. This increases the positive effects on net earnings and utility for all groups of worker (table 8). This shows how inefficient the discontinuity of the minijob policy is.

## 7 Conclusion

We present a simple equilibrium job search model in which firms tailor their offers to the aggregate preferences of job-seekers. As a result, a worker's labor supply choice set depends on other workers' preferences. This represents a channel through which firms mediate responses to tax policies - even when labor costs are not directly affected. We apply our model to a unique setting where we observe strong reactions to a *making work pay* (MWP) policy for workers who are not directly affected. In Germany earnings below a threshold are exempted from income tax and employees' SSC, generating a substantial discontinuity in the budget set. These so-called minijobs aim at increasing employment. In the period of analysis this special tax treatment did not apply for second jobs. Yet, second job earnings feature strong bunching at the threshold, strongly suggestive of the existence of equilibrium effects.

We set-up an equilibrium job search model which allows for a discontinuous tax schedule. We are able to rationalize the discontinuous bunching in the earnings distributions of main and second jobs as firm responses to workers' aggregate preferences. We structurally estimate our model exploiting this specific institutional setting. We estimate the extensive margin effect of the tax exemption: 1.5 million people only take on a low-paid job due to the tax exemption.

We then simulate counter-factual policy reforms which remove the discontinuity induced by the tax exemption: Apart from creating jobs, the tax exemption has substantial negative effects on earnings of individuals who would also work without a subsidy. A first scenario completely removes the tax exemption. This increases earnings of the latter by on average over 50 €/month (ca. 11% of earnings). In a second reform scenario, we replace the minijob tax exemption by a smooth subsidy schedule. This prevents the distortions generated by a discontinuous tax schedule while retaining the positive employment effect. This reform prevents earnings bunching at the threshold, increasing total earnings in the low-paid market by over ten percent and - although the new tax schedule is more generous - increasing total tax revenues as well. While removing the tax exemption generates a group of workers who unambiguously lose utility, our second reform scenario improves welfare for all groups of workers.

Negative effects on earnings of already employed individuals is a usual side effect of

MWP policies like the Earned Income Tax Credit in the US or the Working Family Tax Credit in the UK (Blundell, 2000; Bargain et al., 2010). We show that firms contribute crucially to these earnings effects. In both scenarios that reduce the tax exemption, firms offer more jobs above the threshold than in the status quo. The abundance of job offers at the threshold benefits some while hurting others. In our case individuals with rich spouses, pensioners and students benefit at the cost of unemployed workers seeking any kind of employment. More generally, less dominant groups in the labor market will find it difficult to get acceptable offers if their preferred hours differ strongly from average preferences. Such equilibrium effects might, for example, play a role in reinforcing low levels of labor supply of women when a substantial part of women desire working part-time. However, our model also reveals less obvious reactions: The increase in competitiveness of the labor market incites firms to post high-utility offers below the threshold. When employee tax subsidies are withdrawn, firms respond by *reducing* their earnings offers. Firms' responses represent reactions to changes in labor supply preferences. We thus complement other work which discusses how unintended consequences of making work pay policies unfold in equilibrium (Rothstein, 2010; Leigh, 2010; Kolm and Tonin, 2011; Tazhitdinova, 2017). While these studies focus on channels like incidence and displacement effects, we add another potential channel.

The tax exemption studied here was reformed in 2003. Unfortunately, the negative welfare effects highlighted here remain very relevant: The distortion has not been removed. On the one hand, the reform attenuated the distorting effect of the minijob policy by smoothing the drop in net earnings at the threshold. However, on the other hand, the tax exemption was generalized to include second jobs as well as first jobs. (This makes identification of equilibrium effects less clear, which is why we focus on the period prior to 2003 in our estimation.) The fraction of workers in the market eligible for the tax exemption thus increased, raising the incentive for firms to offer jobs below or at the threshold. Thus the problematic redistributive aspects highlighted here remain.

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# A Derivation of firm-size

We now derive the number of type-$s$ and 0 workers a firm can attract by offering earnings $z$, based on equations 2, 4 and 5. We start with type-$s$ workers:

$$
\begin{aligned}
l^s(z) &= \frac{\frac{F(z)}{1+\kappa^s(1-F(z))} - \frac{F(z-\epsilon)}{1+\kappa^s(1-F(z-\epsilon))}}{F(z)-F(z-\epsilon)}(n^s-u^s) \\
&= \frac{\frac{F(z)(1+\kappa^s(1-F(z-\epsilon)))-F(z-\epsilon)(1+\kappa^s(1-F(z)))}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))}}{F(z)-F(z-\epsilon)}(n^s-u^s) \\
&= \frac{\frac{F(z)+F(z)\kappa^s-F(z)\kappa^sF(z-\epsilon)-(F(z-\epsilon)+F(z-\epsilon)\kappa^s-F(z-\epsilon)\kappa^sF(z))}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))}}{F(z)-F(z-\epsilon)}(n^s-u^s) \\
&= \frac{\frac{F(z)+F(z)\kappa^s-(F(z-\epsilon)+F(z-\epsilon)\kappa^s)}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))}}{F(z)-F(z-\epsilon)}(n^s-u^s) \\
&= \frac{\frac{(F(z)-F(z-\epsilon))(1+\kappa^s)}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))}}{F(z)-F(z-\epsilon)}(n^s-u^s) \\
&= \frac{(1+\kappa^s)}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))}(n^s-u^s) \\
&= \frac{(1+\kappa^s)}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))}\frac{n^s\kappa}{1+\kappa^s} \\
&= \frac{n^s\kappa}{(1+\kappa^s(1-F(z)))(1+\kappa^s(1-F(z-\epsilon)))}
\end{aligned}
$$

type-0 workers do not accept jobs with $z > z^*$. For $z \leq z^*$ we thus get

$$
\begin{aligned}
l^0(z) &= \frac{\frac{F(z)}{(1+\kappa^0(F(z^*)-F(z)))F(z^*)} - \frac{F(z-\epsilon)}{(1+\kappa^0(F(z^*)-F(z-\epsilon)))F(z^*)}}{F(z)-F(z-\epsilon)}(n^0-u^0) \\
&= \frac{\frac{F(z)((1+\kappa^0(F(z^*)-F(z-\epsilon)))F(z^*))-F(z-\epsilon)((1+\kappa^0(F(z^*)-F(z)))F(z^*))}{((1+\kappa^0(F(z^*)-F(z)))F(z^*))((1+\kappa^0(F(z^*)-F(z-\epsilon)))F(z^*))}}{F(z)-F(z-\epsilon)}(n^0-u^0) \\
&= \frac{\frac{(F(z)+F(z)\kappa^0F(z^*)-F(z)\kappa^0F(z-\epsilon)-(F(z-\epsilon)+F(z-\epsilon)\kappa^0F(z^*)-F(z-\epsilon)\kappa^0F(z)))F(z^*)}{F(z^*)^2(1+\kappa^0(F(z^*)-F(z)))(1+\kappa^0(F(z^*)-F(z-\epsilon)))}}{F(z)-F(z-\epsilon)}(n^0-u^0) \\
&= \frac{\frac{(F(z)+F(z)\kappa^0F(z^*)-(F(z-\epsilon)+F(z-\epsilon)\kappa^0F(z^*)))}{F(z^*)(1+\kappa^0(F(z^*)-F(z)))(1+\kappa^0(F(z^*)-F(z-\epsilon)))}}{F(z)-F(z-\epsilon)}(n^0-u^0) \\
&= \frac{\frac{(F(z)-F(z-\epsilon))(1+\kappa^0F(z^*))}{F(z^*)(1+\kappa^0(F(z^*)-F(z)))(1+\kappa^0(F(z^*)-F(z-\epsilon)))}}{F(z)-F(z-\epsilon)}(n^0-u^0) \\
&= \frac{(1+\kappa^0F(z^*))}{F(z^*)(1+\kappa^0(F(z^*)-F(z)))(1+\kappa^0(F(z^*)-F(z-\epsilon)))}(n^0-u^0) \\
&= \frac{(1+\kappa^0F(z^*))}{F(z^*)(1+\kappa^0(F(z^*)-F(z)))(1+\kappa^0(F(z^*)-F(z-\epsilon)))}\frac{n^0\kappa^0F(z^*)}{1+\kappa^0F(z^*)} \\
&= \frac{n^0\kappa^0}{(1+\kappa^0(F(z^*)-F(z)))(1+\kappa^0(F(z^*)-F(z-\epsilon)))}
\end{aligned}
$$

The total firm size is then the sum of the number of workers of each type, which gives expression 6.

# B Sketch of Proof of Propositions (I) - (III)

**Proposition (I)** *If we observe offers above $z^*$, there must be a mass point of job offers at $z^*$. The wage offer distribution above $z^*$ is continuous up to some $\bar{z}$.*

Assume there exists no mass point (i.e. $f(z^*) = 0$), the offer distribution for $z < z^*$ is then continuous and profits at the threshold are

$$\pi(z^*) = (p - z^*)\frac{n^s\kappa^s}{(1 + \kappa^s(1 - F(z^*)))^2} + n^0\kappa^0. \tag{17}$$

Profits associated with offering earnings slightly above the threshold (for $\epsilon \to 0$) are:

$$\begin{aligned}\pi(z^* + \epsilon) &= (p - (z^* + \epsilon))\frac{n^s\kappa^s}{(1 + \kappa^s(1 - F(z^* + \epsilon)))(1 + \kappa^s(1 - F(z^* - \epsilon + \epsilon)))} \\ &= (p - z^*)\frac{n^s\kappa^s}{(1 + \kappa^s(1 - F(z^*)))^2}\end{aligned} \tag{18}$$

Assuming $f(z^*) = 0$ it holds that $\pi(z^* + \epsilon) < \pi(z^*)$ implying that there is a gap to the right of the threshold.

Is there an earnings level $z' > z^* + \epsilon$ where the equal profit condition holds again? Equation 19 makes use of $F(z') = F(z^*)$ which holds if there is a gap in the interval $z \in (z^*, z')$.

$$\begin{aligned}\pi(z') &= (p - z')\frac{n^s\kappa^s}{(1 + \kappa^s(1 - F(z')))(1 + \kappa^s(1 - F(z' - \epsilon)))} \\ &= (p - (z'))\frac{n^s\kappa^s}{(1 + \kappa^s(1 - F(z^*)))^2}\end{aligned} \tag{19}$$

As $(p - z') < (p - z^*)$ it holds that $\pi(z') < \pi(z^*)$ implying that no job with earnings $z > z^*$ will be offered if there is no mass point at $z^*$. Allowing for a mass point at $z^*$, $\frac{\partial\pi(z^*)}{\partial f(z^*)} < 0$ and $\frac{\partial\pi(z^*+\epsilon)}{\partial f(z^*)} = 0$ imply that there might be a value for $f(z^*)$ for which the equal profit condition between $z^*$ and $z^*+\epsilon$ holds ($\pi(z^*+\epsilon) = \pi(z^*)$). For earnings $z' \in [z^*+\epsilon, \overline{z}]$, the usual trade-off between profit per workers and firm size ensures that the equilibrium offer distribution is continuous in that interval and determined by $\pi(z') = \pi(z^* + \epsilon)$.

**Proposition (II)** *If there is a mass point at $z^*$, there will be a gap in the offer distribution just below the threshold. There may or may not exist wage offers below the threshold*

*$z^*$ in equilibrium. The wage offer distribution will then be continuous between $z \in [\underline{z}, z'']$ for $z'' < z^*$.*

We first need to show that a mass point in the wage offer distribution is only consistent with equal profits if there is a gap in the wage offering distribution. We compare profits $\pi(z^*)$ with profits $\pi(z^* - \epsilon)$. Profits of a job offer with earnings at the threshold are given by

$$
\begin{aligned}
\pi(z^*) &= (p - z^*)l(z^*) \\
&= (p - z^*)\left(\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)))(1+\kappa^s(1-F(z^*-\epsilon)))} + \frac{n^0\kappa^0}{(1+\kappa^0(F(z^*)-F(z^*)))(1+\kappa^0(F(z^*)-F(z^*-\epsilon)))}\right) \\
&= (p - z^*)\left(\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)))(1+\kappa^s(1-F(z^*)+f(z^*)))} + \frac{n^0\kappa^0}{(1+\kappa^0 f(z^*))}\right) \qquad (20)
\end{aligned}
$$

Profits slightly below the threshold are given by (for $\epsilon \to 0$):

$$
\begin{aligned}
\pi(z^* - \epsilon) = (p - (z^* - \epsilon))(&\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*-\epsilon)))(1+\kappa^s(1-F(z^*-2\epsilon)))} + \\
&\frac{n^0\kappa^0}{(1+\kappa^0(F(z^*)-F(z^*-\epsilon)))(1+\kappa^0(F(z^*)-F(z^*-2\epsilon)))}) \\
&= (p - z^*)(\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*-\epsilon)))^2} + \frac{n^0\kappa^0}{(1+\kappa^0(f(z^*)))^2}) \\
&= (p - z^*)(\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)+f(z^*)))^2} + \frac{n^0\kappa^0}{(1+\kappa^0(f(z^*)))^2})
\end{aligned}
$$

Given proposition (I), the data implies that there is a mass point at $z = z^*$ (i.e. that $f(z^*) > 0$). When $f(z^*) > 0$, it holds that $(1+\kappa^j(1-F(z^*)+f(z^*))) > (1+\kappa^j(1-F(z^*)))$ and $(1 + \kappa^0(f(z^*))) > 1$. Therefore, $\pi(z^*) > \pi(z^* - \epsilon)$ and there will be a gap to the left of the threshold.

For the second part of proposition (II), define the highest wage offer below the threshold as $z''$, such that $F(z'') = F(z^* - \epsilon)$. Note that since there is a gap left of the threshold, if in equilibrium a $z''$-offer exists, it may be significantly below $z^*$. In equilibrium any $z''$-offer must make the same amount of profits as the threshold wage offer $z^*$.

$$
\begin{aligned}
\pi(z'') = (p - z'')&\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z'')))(1+\kappa^s(1-F(z''-\epsilon)))} + \\
&\frac{n^0\kappa^0}{(1+\kappa^0(F(z^*)-F(z'')))(1+\kappa^0(F(z^*)-F(z''-\epsilon)))} \\
&= (p - z'')\frac{n^s\kappa^s}{(1+\kappa^s(1-F(z^*)+f(z^*)))^2} + \frac{n^0\kappa^0}{(1+\kappa^0(f(z^*)))^2} \qquad (21)
\end{aligned}
$$

Comparing equations 20 and 21 illustrates that $\pi(z'') = \pi(z^*)$ can hold as $\pi(z'')$ increases with decreasing $z''$. That is, there might be a certain size of the gap where $\pi(z'') = \pi(z^*)$ holds.

Using that $F(\underline{z}) = 0$, we now determine the lowest wage offer $\underline{z}$ that will be made (if there are wage offers below $z^*$).

$$\pi(\underline{z}) = (p-\underline{z})\frac{n^s\kappa^s}{(1+\kappa^s(1-F(\underline{z})))(1+\kappa^s(1-F(\underline{z}-\epsilon)))}+ \\ \frac{n^0\kappa^0}{(1+\kappa^0(F(z^*)-F(\underline{z})))(1+\kappa^0(F(z^*)-F(\underline{z}-\epsilon)))} \\ = (p-\underline{z})\frac{n^s\kappa^s}{(1+\kappa^s)^2}+\frac{n^0\kappa^0}{(1+\kappa^0(F(z^*)))^2} \tag{22}$$

If $z'' < z^*$ exists and $\underline{z} < z''$ there will then be a continuity of wage offers between these two values, generating equal profits with the standard trade-off between margins and firm-size.

# C Discrete variation in working hours

In the main text, we assume a continuous variation in hours worked. We here now discuss the model equilibrium with discrete variation in hours. For simplification, we allow for two different numbers of hours $h_k$ where $k \in (1,2)$. In the market for low-paid jobs, this may correspond to 10 and 20 hours of working. We later consider how this model generalizes to three or more hours sectors.

Firms set wage rates, $w$, workers care about wage rates as well as hours. In order to simplify notation, we follow Shephard (2017) and define $q_2(w) = w$ and $U(q_1(w), h_1) = U(q_2(w), h_2) = U(w, h_2)$, so $q_1(w)$ is a function that denotes the wage rate that makes individuals indifferent between working with few ($k = 1$) hours at $q_1(w)$ or working more ($k = 2$) hours at $w$. Depending on preferences, individuals may require a low-hours premium or accept a low-hours wage penalty.

## C.1 Worker mobility

The flow into and out of small jobs is equivalent to before (see equation 2). The flow of workers of type $j \in (s, 0a, 0m)$ from and into jobs with hours $h_k$ and wage rate $w$ is

$$D^j(w)g_k^j(q_k(w))e_k^j = \lambda^j f_k(q_k(w))(u^j + G_1^j(q_1(w)-\epsilon)e_1^j + G_2^j(w-\epsilon)e_2^j) \tag{23}$$

with $D^j(w) = [\delta + \lambda^j((1-F_2(w)) + (1-F_1(q_1(w))))]$ for $j \in (s, 0a)$. Equation 24 states the corresponding definition for workers of type $0m$ who do not accept jobs with wage rates larger than $w_k^*$.

$$D^{0m}(w) = \begin{cases} [\delta^0 + \lambda^0((F_2(w_2^*)-F_2(w)) + (F_1(w_1^*)-F_1(q_1(w))))] & \forall w \le w_2^* \text{ and } q_1(w) \le w_1^* \\ [\delta^0 + \lambda^0(F_2(w_2^*)-F_2(w))] & \forall w \le w_2^* \text{ and } q_1(w) > w_1^* \end{cases} \tag{24}$$

The LHS of equation 23 pertains to workers who leave a job in a sector $k$ with wage $q_k(w)$. For $k = 2$ this group consists of workers who move from sector 2 to sector 1 ($\lambda^j(1 - F_1(q_1(w)))g_2^j(w)e_2^j$ for $j \in (s, 0a)$), who move to a better paying job within sector 2 ($\lambda^j(1 - F_2(w))g_2^j(w)e_2^j$ for $j \in (s, 0a)$) and who lose their small job ($\delta^j g_2^j(w)e_2^j$). The RHS pertains to workers who start a job in sector $k$ with wage rate $q_k(w)$. For $k = 2$ this consists of workers who move from sector 1 to sector 2 ($\lambda^j f_2(w)G_1^j(q_1(w) - \epsilon)e_1^j$), who changes jobs within sector 2 ($\lambda^j f_2(w)G_2^j(w - \epsilon)e_2^j$) and who were previously job-seeking ($\lambda^j f_2(w)$). The overall flow (i.e. both sectors) due to separations from jobs with wage rate of no greater than $w$ is:

$$\begin{aligned}(G_1^j(q_1(w))e_1^j + G_2^j(w)e_2^j)D^j(w) &= \lambda^j u^j F_1(q_1(w)) + \lambda^j u^j F_2(w)\\ &= \lambda^j u^j + \lambda^j u^j - \lambda^j u^j(1 - F_1(q_1(w))) - \lambda^j u^j(1 - F_2(w)) \qquad (25)\end{aligned}$$

Using equation 2 gives

$$G_1^j(q_1(w))e_1^j + G_2^j(w)e_2^j = \frac{\delta^j n^j - u^j D^j(w)}{D^j(w)}. \qquad (26)$$

By combining equations 23 and 26 we obtain

$$g_k^j(q_k(w))e_k^j = \frac{\lambda^j f_k(q_k(w))(u^j + \frac{\delta^j n^j - u^j(D^j(w-\epsilon))}{D^j(w-\epsilon)}}{D^j(w)} \qquad (27)$$

## C.2 Firm size

The number of workers of type $j$ in steady-state employed at a firm in sector $k$ which offers wage rate $q_k(w)$ is

$$\begin{aligned}l_k^j(q_k(w)) &= \frac{g_k^j(q_k(w))e_k^j}{f_k(q_k(w))}\\ &= \frac{\lambda^j \delta n^j}{D^j(w)D^j(w-\epsilon)}. \qquad (28)\end{aligned}$$

The steady state firm size is then

$$\begin{aligned}l_k(q_k(w)) &= l_k^s(q_k(w)) + l_k^{0a}(q_k(w)) + l_k^{0m}(q_k(w))\\ &= \begin{cases} \frac{\lambda^s\delta^s n^s}{D^s(w)D^s(w-\epsilon)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(w)D^{0a}(w-\epsilon)} + \frac{\lambda^0\delta^0 n^{0m}}{D^{0m}(w)D^{0m}(w-\epsilon)} & \forall w \leq w^* \\ \frac{\lambda^s\delta^s n^s}{D^s(w)D^s(w-\epsilon)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(w)D^{0a}(w-\epsilon)} & \forall w > w^* \end{cases}. \qquad (29)\end{aligned}$$

Following the standard arguments of profit equalization, we find the following (the reasoning is parallel to the case without hours variation):

**Proposition (A1)** *There can be (at most one) mass point in the wage offer distribution at the threshold in each sector, i.e. at wages* $w_k^* \equiv \frac{z^*}{h_k}$.

**Sketch of Proof**: The following argument closely mirrors the argument in the case of homogeneous hours. We compare profits at the threshold value with profits above. We find that if there exist offers above, there must be a mass point at the threshold.

The profit of a sector $k$ firm offering wage rate $q_k(w)$ can be expressed as $\pi_k(q_k(w)) = (ph_k - q_k(w)h_k)l_k(q_k(w))$. We first state the profits of a type-2-firm, assuming that $q_1(w_2^*) \leq w_1^*$.

$$\begin{aligned}
\pi_2(w_2^*) &= \frac{\lambda^s\delta^s n^s}{D^s(w_2^*)D^s(w_2^*-\epsilon)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(w_2^*)D^{0a}(w_2^*-\epsilon)} + \frac{\lambda^0\delta^0 n^{0m}}{D^{0m}(w_2^*)D^{0m}(w_2^*-\epsilon)} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(w_2^*))+(1-F_1(q_1(w_2^*))))][\delta^s+\lambda^s((1-F_2(w_2^*-\epsilon))+(1-F_1(q_1(w_2^*)-\epsilon)))]}+ \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(w_2^*))+(1-F_1(q_1(w_2^*))))][\delta^0+\lambda^0((1-F_2(w_2^*-\epsilon))+(1-F_1(q_1(w_2^*)-\epsilon)))]}+ \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0(F_1(w_1^*)-F_1(q_1(w_2^*)))][\delta^0+\lambda^0(f_2(w_2^*)+(F_1(w_1^*)-F_1(q_1(w_2^*)-\epsilon)))]}
\end{aligned} \tag{30}$$

Evaluated marginally above the threshold, profits are

$$\begin{aligned}
\pi_2(w_2^*+\epsilon) &= \frac{\lambda^s\delta^s n^s}{D^s(w_2^*+\epsilon)D^s(w_2^*)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(w_2^*+\epsilon)D^{0a}(w_2^*)} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(w_2^*+\epsilon))+(1-F_1(q_1(w_2^*+\epsilon))))][\delta^s+\lambda^s((1-F_2(w_2^*))+(1-F_1(q_1(w_2^*))))]}+ \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(w_2^*+\epsilon))+(1-F_1(q_1(w_2^*+\epsilon))))][\delta^0+\lambda^0((1-F_2(w_2^*))+(1-F_1(q_1(w_2^*))))]}
\end{aligned} \tag{31}$$

Equations 30 and 31 show that the equal profit condition can only hold if there is a mass point in the offer distribution of sector 2 at $w_2^*$. By symmetry, note that the same argument can be made with respect to a type-1 firm. However, if the utility of a threshold offer lies in the "gap area" due to a threshold in another sector, it may be the case that there is no mass point in that sector. This explains the restriction "at most one" in Proposition (IV) and completes our discussion.

We now consider the influence of thresholds in other hours sectors on the wage distribution. Consider a firm of type 2, i.e. seeking a worker to work for $h_2$ hours. The impact of a potential mass point in the offer distribution of sector 1 at $w_1^*$ depends on the relation between $w_2^*$, $q_1(w_2^*)$ and $w_1^*$.

**Proposition (A2)** *There will be no wage offers at wage levels (and in a certain interval below this level) that offer the same utility as is available at threshold wages* $w^*_{j\neq\ k}$ *in other sectors.*

The intuition for Proposition (A2) is the following: It is a dominated strategy to offer a wage rate that is equal in utility to an offer made by several other firms. A slightly higher offer will attract all workers from these firms at only marginal cost. By Proposition (IV), wage offers at earnings thresholds generate mass points in the wage offer distributions. Thus for example a type-2 firm will offer a wage rate slightly larger than $\tilde{w}_2$ (where $U(\tilde{w}_2, h_2) = U(w^*_1, h_1)$.) in order to additionally attracts workers from this positive mass of sector 1 firms. This implies that there must be a gap in the wage offer distribution at $\tilde{w}_2$. How much below this utility value an offer can be sustained in equilibrium will depend on the parameters of the model in an analogous way to the potential existence of offers below the threshold offer in the homogeneous case.

**Sketch of proof**: Let $\tilde{w}_2$ denote the wage rate which satisfies $U(\tilde{w}_2, h_2) = U(w^*_1, h_1)$. If $\tilde{w}_2 > w^*_2$ the profits of a sector 2 firm offering wage rate $\tilde{w}_2$ and slightly above are:

$$\begin{aligned}
\pi_2(\tilde{w}_2) &= \frac{\lambda^s \delta n^s}{D^s(\tilde{w}_2)D^s(\tilde{w}_2-\epsilon)} + \frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(\tilde{w}_2)D^{0a}(\tilde{w}_2-\epsilon)} + \frac{\lambda^0\delta^0 n^{0m}}{D^{0m}(\tilde{w}_2)D^{0m}(\tilde{w}_2-\epsilon)} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w^*_1)))][\delta^s+\lambda^s((1-F_2(\tilde{w}_2-\epsilon))+(1-F_1(w^*_1-\epsilon)))]} + \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w^*_1)))][\delta^0+\lambda^0((1-F_2(\tilde{w}_2-\epsilon))+(1-F_1(w^*_1-\epsilon)))]} + \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w^*_2)-F_2(\tilde{w}_2))+(F_1(w^*_1)-F_2(w^*_1))))][\delta^0+\lambda^0((F_2(w^*_2)-F_2(\tilde{w}_2-\epsilon))+(F_1(w^*_1)-F_2(w^*_1-\epsilon)))]} \\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w^*_1)))][\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w^*_1)+f_1(w^*_1))))]} + \\
&+ \frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w^*_1)))][\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w^*_1)+f_1(w^*_1)))]} + \\
&+ \frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w^*_2)-F_2(\tilde{w}_2)))][\delta^0+\lambda^0((F_2(w^*_2)-F_2(\tilde{w}_2))+f_1(w^*_1))]}
\end{aligned} \tag{32}$$

$$
\begin{aligned}
\pi_2(\tilde{w}_2-\epsilon) &= \frac{\lambda^s\delta^s n^s}{D^s(\tilde{w}_k-\epsilon)D^s(\tilde{w}_k-2\epsilon)}+\frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(\tilde{w}_2-\epsilon)D^{0a}(\tilde{w}_2-2\epsilon)}+\frac{\lambda^0\delta^0 n^{0m}}{D^{0m}(\tilde{w}_2-\epsilon)D^{0m}(\tilde{w}_2-2\epsilon)}\\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2-\epsilon))+(1-F_1(w_1^*-\epsilon)))][\delta^s+\lambda^s((1-F_2(\tilde{w}_2-2\epsilon))+(1-F_1(w_1^*-2\epsilon)))]}+\\
&+\frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2-\epsilon))+(1-F_1(w_1^*-\epsilon)))][\delta^0+\lambda^0((1-F_2(\tilde{w}_2-2\epsilon))+(1-F_1(w_1^*-2\epsilon)))]}+\\
&+\frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2-\epsilon))+(F_1(w_1^*)-F_2(w_1^*-\epsilon)))][\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2-2\epsilon))+(F_1(w_1^*)-F_2(w_1^*-2\epsilon)))]}\\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)+f_1(w_1^*)))]^2}+\\
&+\frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)+f_1(w_1^*)))]^2}+\\
&+\frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2))+f_1(w_1^*))]^2}
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
\pi_2(\tilde{w}_2+\epsilon) &= \frac{\lambda^s\delta^s n^s}{D^s(\tilde{w}_2+\epsilon)D^s(\tilde{w}_2)}+\frac{\lambda^0\delta^0 n^{0a}}{D^{0a}(\tilde{w}_2+\epsilon)D^{0a}(\tilde{w}_2)}+\frac{\lambda^0\delta^0 n^{0m}}{D^{0m}(\tilde{w}_2+\epsilon)D^{0m}(\tilde{w}_2)}\\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2+\epsilon))+(1-F_1(w_1^*+\epsilon)))][\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]}+\\
&+\frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2+\epsilon))+(1-F_1(w_1^*+\epsilon)))][\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]}+\\
&+\frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2+\epsilon)))][\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2)))]}\\
&= \frac{\lambda^s\delta^s n^s}{[\delta^s+\lambda^s((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]^2}+\\
&+\frac{\lambda^0\delta^0 n^{0a}}{[\delta^0+\lambda^0((1-F_2(\tilde{w}_2))+(1-F_1(w_1^*)))]^2}+\\
&+\frac{\lambda^0\delta^0 n^{0m}}{[\delta^0+\lambda^0((F_2(w_2^*)-F_2(\tilde{w}_2)))]^2}
\end{aligned}
\tag{34}
$$

As $f_1(w_1^*) > 0$ and $\epsilon \to 0$, it holds that $\pi_2(\tilde{w}_2-\epsilon) < \pi_2(\tilde{w}_2) < \pi_2(\tilde{w}_2+\epsilon)$. This implies that there will be no wage offers of value $\tilde{w}_2$. As $(ph - wh)$ increases with decreasing w, there might be a wage rate $w'$ where it holds that $\pi_2(w') = \pi_2(\tilde{w}_2+\epsilon)$. This implies that $f_2(.)$ exhibits a gap in the interval (w',$\tilde{w}_2$]. If $\tilde{w}_2 < w_2^*$, the terms in equations 32 and 34 referring to workers of type $0m$ drop out. Although this might reduce the extent of the gap, $\pi_2(\tilde{w}_2) < \pi_2(\tilde{w}_2+\epsilon)$ still holds. If $\tilde{w}_2 = w_2^*$ the necessary size of the mass point at $w_2^*$ to balance the loss of type-$0m$ workers decreases (in comparison to $\tilde{w}_2 \neq w_2^*$). How large the gap is, i.e. whether any offers will be made below $\tilde{w}_2$ will depend on the economic environment captured by the parameters of the model.

# D Predicting income tax rates

In Germany, income tax rates depend on household characteristics, but the SIAB data set used in this analysis does not include these. We thus impute tax rates based on another data set, the GSOEP. The tax simulation accounts for the most important aspects of the

German tax-transfer system (Junge, 2017). For type-0 workers, explaining tax rates using variables included in both data sets does not yield satisfying predictions. As we aim at predicting the marginal tax rate directly above the threshold, gross earnings, the most valuable predictor vary only marginally. Other variables hardly add explanatory power (Junge, 2017). We therefore replicate the SIAB sample in the GSOEP data and differentiate three tax groups based on the distribution of simulated tax rates. Observations in the estimation sample are then allocated randomly the mean tax rate of one of these groups. Since earnings at the threshold are less than the general tax allowance, almost 50% of type-0 workers have an income tax rate of zero (table 9). The imputed tax rate for the next 40 % is about eight per cent, for the highest decile it is approximately 20 %.

Tax rates for type-$s$ workers are explained by a Tobit model with first job earnings and sex as explaining variables (table 10).[30] Based on the same information in the SIAB data we then predict the individual tax rates. Type-$s$ workers are also allocated to three groups (table 9). Tax rates are higher for type-$s$ workers as they already have first job earnings. As our sample is restricted to observations with first job earnings of more than 1000 € (section 4.1), all observations exceed the general tax allowance and thus have a strictly positive marginal tax rate.

**Table 9:** Income tax groups

| | Type s | | | Type 0 | | |
|---|---|---|---|---|---|---|
| Group | % | t | std | % | t | std |
| 1 | 25 | 8.32 | 1.75 | 48.27 | 0 | 0 |
| 2 | 65 | 12.99 | 4.17 | 41.73 | 7.92 | 4.64 |
| 3 | 10 | 27.6 | 8.52 | 10 | 19.47 | 4.50 |

*Notes:* Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0 workers have or seek a low-paid job but have no other job; t= mean average income tax rate at 326 €; std=standard deviation; Type-$s$: aggregated predicted tax rates based on model in table 10; type-0: aggregated observed tax rates
*Source:* Own calculations based on GSOEP wave 1999-2002

**Table 10:** Estimation results: tax rates of type-$s$ workers

| | coeff. | s.e. |
|---|---|---|
| Male | -0.0422 | 0.003 |
| Yearly first job earnings | 4.27e-06 | 7.50e-08 |
| Constant | 0.0354 | 0.002 |

*Notes:* coeff.=regression coefficient, s.e.=standard errors.
*Source:* Own calculations based on GSOEP wave 1999-2002

[30] To increase precision we use all second jobs for the estimation of tax rates. We do not restrict the sample with respect to first job earnings as done for the main estimation.

# E Predicting hours of work

We do not have information on hours of work offered in the market. We, therefore, seek to impute actual working hours by means of survey data (GSOEP). To do that we model hours based on variables included in both data sets: sex, gross earnings, sector and education. As the hours distribution resembles a log-normal distribution (see figure 3), a generalized linear model with log link is used for imputation. As in the main analysis the estimation sample comprises two groups of employees. These are, first, employees with only one job and earnings of less than 800 €/month. We use their actual hours of work per week. The sample includes, second, employees who additionally hold a full-time job paying at least 1000 €/month and a side job with earnings below 800 €/month. Weekly hours of work in the side job are used as dependent variable. The model is estimated jointly for both groups (table 11). The resulting hours distribution increases sharply up to a global peak at approximately 50 hours per month (figure 7). The right tail covers hours up to 200. The mean is fit very well. The variation is however smaller in the predicted data (table 12).

**Table 11:** Estimation results: hours

| | coeff. | s.e. |
|---|---|---|
| Gross Earnings | 0.00 | 0.00 |
| Sex | 0.03 | 0.05 |
| *Sector* | | |
| Manufacturing | -0.38 | 0.11 |
| Energy, Water | -0.28 | 0.20 |
| Construction | -0.21 | 0.14 |
| Wholesale and retail | -0.26 | 0.10 |
| Hotels and restaurants | -0.23 | 0.15 |
| Transport | -0.40 | 0.14 |
| Finance | -0.43 | 0.17 |
| Real Estate | -0.44 | 0.11 |
| Public Admin | -0.51 | 0.14 |
| Education | -0.48 | 0.13 |
| Health | -0.57 | 0.11 |
| Other Services | -0.28 | 0.12 |
| Households | -0.35 | 0.14 |
| Other Sector | -0.33 | 0.11 |
| *Education* | | |
| Other Education | 0.56 | 0.13 |
| Basic | 0.26 | 0.09 |
| Middle Voc. | 0.28 | 0.09 |
| Higher Voc. | 0.18 | 0.10 |
| Constant | 1.83 | 0.14 |

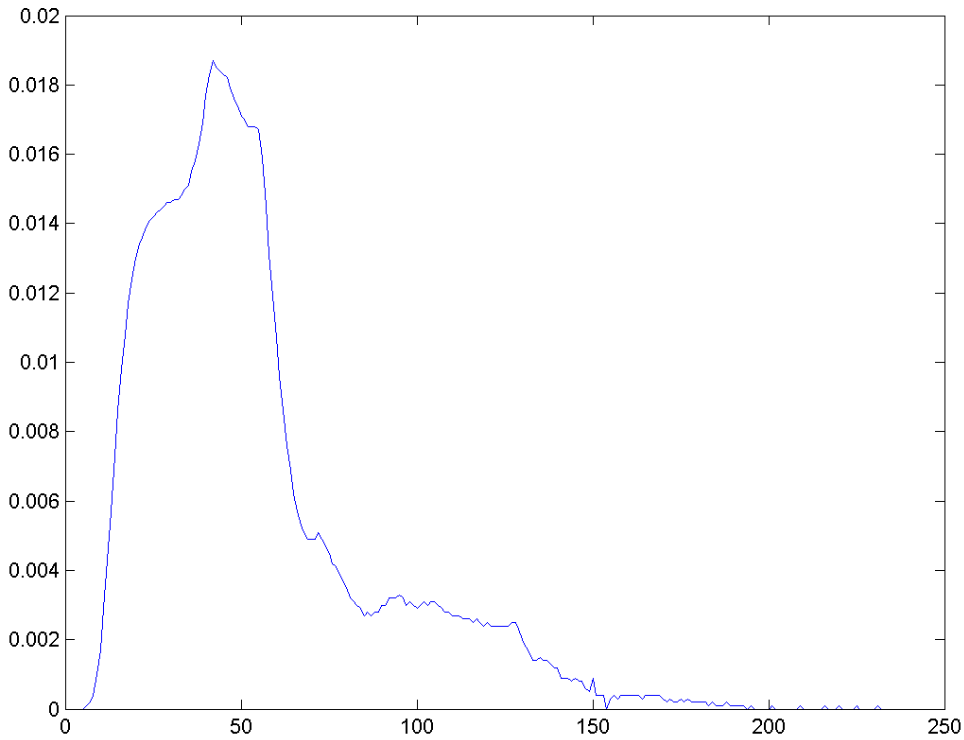
*Notes:* coeff.=regression coefficient, s.e.=standard errors., dependent variable: weekly hours
*Source:* Own calculations based on GSOEP wave 2001

**Table 12:** Fit of hours distribution

| | Observed | Predicted |
|---|---|---|
| *mean* | 57.25 | 57.14 |
| $p(25)$ | 23.11 | 34.45 |
| $p(50)$ | 43.33 | 52.17 |
| $p(75)$ | 86.67 | 76.91 |

*Notes:* $p(x)$: xth quantile
*Source:* SIAB; own calculations.

**Figure 7:** Probability density function of monthly hours of work




*Notes:* Predicted hours distribution based on a generalized linear model with log link and an estimation sample as in the main estiamtion (section 4).
*Source:* SOEP, SIAB, own calculations.

# F Numerical approaches

This section describes two numerical approaches applied in the estimation procedure. First, the numerical calculation of the job offer distribution $F(.)$ and, second, the numerical optimization of the likelihood.

## F.1 Job offer distribution

In the case with homogeneous hours the offer distribution can be characterized by a system of equal profit conditions. Firms with different hours, however, may make different profits in equilibrium. The offer distribution in the case of heterogeneous hours is, thus, calculated by solving the maximization problem successively for all firms in the market. Firms take the current offer distribution as given which is then updated once a firm has decided on a wage offer. The order of firms deciding on an offer is determined by random sampling with replacement. We run the algorithm based on 100 firms which first draw once from the predicted hours distribution (see above). Firms can chose between 100 different wages. The algorithm is stopped after 1000 draws. After this, the distribution is fairly stable: The estimated fraction of offers with earnings below or at the threshold, for example, has a standard deviation of less than 0.02 when the algorithm is repeated ten times. To increase precision, we take the average offer distribution of ten repetitions.

### F.2 Optimization of likelihood

The likelihood is optimized with respect to $\theta$, $\alpha$ and $\sigma$ by a two-step grid search procedure (Hansen, 2016). Gradient-based approaches are inappropriate because the costly calculation of the job offer distribution has to be repeated every iteration. It further contains random elements rendering the likelihood function non-smooth.

For $\theta$ and $\alpha$ we evaluate the likelihood in the first step at 11 equally spaced grid points in the interval $[0, 1]$. In the second step the grid is narrowed down to steps of 0.02. For $\sigma$ we use the 50, 100 and 150 as grid points in the first step. The grid is narrowed down to steps of 20 in the second step.

## G Robustness test: Extensive margin reactions

Extensive margin reactions in our model are exclusively driven by type-$0m$ workers who leave the market when the tax exemption is disposed. Due to the assumption of exogenous and constant frictional parameters, the overall number of vacancies posted by firms decreases. We now perform a robustness test in this respect.

In this alternative scenario, type-$0m$ workers are substituted by workers of other types. We model this by assuming that the number of vacancies per period posted by firms stay constant. Offer arrival rates for type-$s$ and $0a$ workers are thus increased. Keeping the ratio between both constant, this implies $\lambda^s = 0.0513$ and $\lambda^0 = 0.3908$. This is comparable to introducing a matching function, accounting for on-the-job search and differing arrival rates across worker groups.

Both type-$s$ and $0a$ workers can substantially increase their net earnings and utility - albeit the latter forgoes the tax exemption. Total hours still decrease by about 12%. This is overcompensated by an increase in average wage rate, though, such that total gross earnings increase. Tax revenues increase accordingly. The results of such a simulation can be interpreted as an upper bound. When economic incidence of the tax exemption, for example, is not entirely on employees, firms profit from lower wage rates for minijobs relative to jobs subject to full SSC. Removing the tax exemption might then lead to a decrease in vacancies. The substantial differences between both scenarios imply that labor demand responses are crucial for comprehensively evaluating the minijob regulation. Our model indeed includes firms catering to the aggregate preferences in the market. It, however, abstracts from other important margins of firm responses.

**Table 13:** Removing the tax and SSC exemption - upper bound scenario

| Change in | Total | type-$s$ | type-0 | type-$0a$ |
|---|---|---|---|---|
| $\bar{w}$ | 0.78 | 0.09 | 0.92 | . |
| $\bar{h}$ | 18.05 | 4.98 | 9.92 | . |
| $\sum h$ | -10.01 | 41.27 | 15.15 | -100 |
| $\bar{z}$ | 179.98 | 36.67 | 156.08 | . |
| $\sum z$ | 9.72 | 47.45 | 41.38 | -100 |
| $\bar{c}$ | 83.96 | 24.98 | 70.31 | . |
| $\bar{v}$ | 69.11 | 20.25 | 55.81 | -199.34 |
| jobs | -1360.5 | 163 | 120.5 | -1644 |
| taxes | 341.04 | 39.64 | 387.61 | -86.21 |

*Notes:* Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-$0a$ workers have or seek a low-paid job but have no other job and would work even when fully taxed. Type-$0m$ workers have or seek a low-paid job but have no other job and would only work when not taxed; $\bar{j}$ – average of variable $j$, $w$ – hourly wage rate, $z$ – monthly gross earnings, $c$ – monthly net earnings, $h$ – weekly hours of work; $\bar{w}$, $\bar{z}$ and $\bar{c}$ in €; $\sum h$ and $\sum z$ in %; jobs in 1000s; taxes in million €; jobs and taxes extrapolated to population; $\bar{w}$, $\bar{h}$, $\bar{z}$, and $\bar{c}$ conditional on employment, $\bar{v}$ based on all individuals
*Source:* SIAB; own calculations.

# H Low-paid market with one worker type

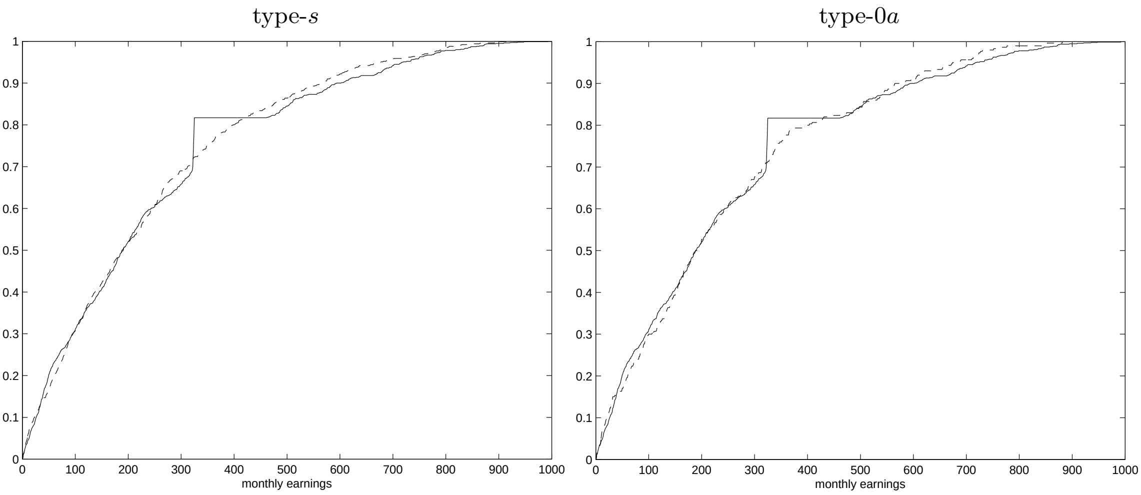
Imagine all workers in the low-paid market are of type $0a$. That is, all workers are eligible for a potential tax exemption but would work when subject to full taxes. Counter-factual simulations in such a market are insightful because they abstract from differential changes of incentives between worker types and compositional changes in the workforce.

When the tax exemption is removed, firms which offered jobs at the threshold in the status quo now mostly offer jobs with higher earnings (left panel of figure 8). This is not surprising as jobs with high earnings are relatively more attractive when they are not subject to a disadvantaged tax treatment. Some firms reduce their earnings offer, though.

The likelihood of earnings farther below the threshold is not affected. Removing the tax exemption has a positive effect on average (and total) gross earnings (table 14). One reason is the higher likelihood of a high earnings offer, another that workers more likely accept a high earnings offer.

Another way to smooth the minijob discontinuity is a gradually increasing tax rate for earnings between 325 and 800 €. The full tax exemption below 325 € remains. Firms which offered jobs at the threshold in the status quo increase their earnings offers. Relative to the status quo the distribution above the threshold is compressed, though. Firms account for the change in workers' incentives due to the high implicit marginal tax rate introduced by the reform.

**Figure 8:** Cumulative earnings offer distribution - status quo vs. reform - only type 0a workers




*Notes:* Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0$a$ workers have or seek a low-paid job but have no other job and would work even when fully taxed. The minijob threshold is at 325€/month. The solid line represents the status quo, the dashed line the counter-factual. *Source:* own calculations.

# I Robustness analysis with respect to market definition

Classifying individuals as job seeking in the main text underlies several assumptions. A spell needs to fulfill two conditions to be classified as type-$s$. First, the respective individual needs to be employed with earnings exceeding 1000 Euro/month. Second, the individual must have had at least one low-paid second job within the analysis period (that is April 1999 to February 2002). On the one hand we disregard individuals seeking for a second job who haven't had one in the analysis period (implying that they have not been able to find one). On the other hand we might mis-classify some individuals as seeking a second job. The large number of job-seeking type-$s$ workers suggest the latter may

**Table 14:** Effects of smoothing the notch - only 0a workers

| Change in | Removal | Kink |
|---|---|---|
| $\bar{w}$ | 0.50 | 0.33 |
| $\bar{h}$ | -0.56 | -2.60 |
| $\sum h$ | -0.58 | -2.71 |
| $\bar{z}$ | 40.73 | 6.93 |
| $\sum z$ | 8.67 | 1.48 |
| $\bar{c}$ | -10.56 | 25.28 |
| $\bar{v}$ | -7.67 | 17.43 |
| jobs | 0.00 | 0.00 |
| taxes | 269.64 | -76.41 |

*Notes:* Removal: Complete removal of tax exemption; Kink: Notch replaced by kink; Type-0$a$ workers have or seek a low-paid job but have no other job and would work even when fully taxed; $\bar{j}$ – average of variable $j$, $w$ – hourly wage rate, $z$ – monthly gross earnings, $c$ – monthly net earnings, $h$ – weekly hours of work; $\bar{w}$, $\bar{z}$ and $\bar{c}$ in €; $\sum h$ and $\sum z$ in %; jobs in 1000s; taxes in million €; jobs and taxes extrapolated to population; $\bar{w}$, $\bar{h}$, $\bar{z}$, and $\bar{c}$ conditional on employment, $\bar{v}$ based on all individuals
*Source:* SIAB; own calculations.

be particularly important. In this section we further restrict the market of second jobs to spells of individuals with a future low-paid second job spell (which starts within the analysis period). The number of job-seeking spells of type-$s$ workers decreases by 10,000 to less than 40,000 (table 15). Similarly, we classify formal unemployment spells as type-0 when the respective individual have had a low-paid job spell in the analysis period. Not all low-paid job spells, however, are preceded by a formal unemployment spell. In this section we additionally classify individuals as small job-seeking all those who are out of labor force. Spells of being out of labor force are specified as the gap between two observed spells of formal (un)employment. The amount of type-0 job-seekers more than doubles to 116,182 (table 15).

**Table 15:** Total number of spells by type and employment - alternative definition of market

| Status \ Type | s | 0 |
|---|---|---|
| Job-seeking | 38,048 | 116,182 |
| Employed in small job | 30,121 | 287,255 |

*Notes*: Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0 workers have or seek a small low-paid job as an only job.

*Source:* SIAB; own calculations

As expected the offer arrival rate of type-0 workers is estimated to be considerably lower than in the main text. By contrast, the offer arrival rate of type-$s$ workers is almost constant. The value of $\theta$ appears rather sensitive and almost halves vis-à-vis the basic specification. The amount of jobs created by the minijob regulation should thus not be over-interpreted.

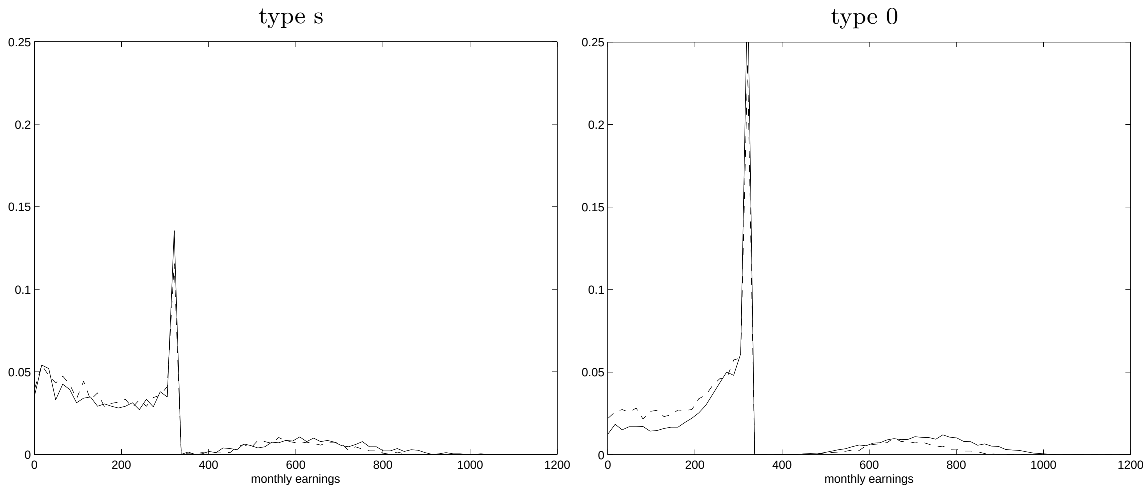
**Table 16:** Parameter estimates - alternative definition of market

| Parameter | p.e. | s.e. |
|---|---|---|
| $\delta^s$ | 0.0474 | 0.0005 |
| $\delta^0$ | 0.0451 | 0.0002 |
| $\lambda^s$ | 0.0296 | 0.0002 |
| $\lambda^0$ | 0.1571 | 0.0028 |
| $\theta$ | 0.2064 | 0.0775 |
| $\alpha$ | 0.8843 | 0.1121 |
| $\sigma$ | 67.500 | 6.963 |
| $p$ | 8 | - |

*Notes:* p.e. – mean point estimate of 50 bootstrap repetitions, s.e. – standard deviation of 50 bootstrap repetitions, $\lambda^j$ – arrival rate for worker type $j$, $\delta^j$ – job destruction rate of worker type $j$, $\alpha$ – relative weight of consumption and leisure in the utility function, $\theta$ – fraction of type-0 workers who do not accept jobs with earnings exceeding the threshold, $\sigma$ – standard deviation of optimization error, $p$ – level of productivity per hour
*Source:* SIAB; own calculations.

The predicted realized earnings distributions are very similar, though (figure 9). For type-0 workers there is slightly more mass to the left of the cap and slightly less at and to the right of the threshold.

**Figure 9:** Predicted realized earnings distribution - basic and alternative distribution




*Notes:* Type-$s$ workers have or seek a low-paid job on top of a job subject to SSC. Type-0 workers have or seek a low-paid job but have no other job. The minijob threshold is at 325€/month. Solid curve pertains to basic specification, dashed curve to alternative specification. Specifications differ in the definition of the labor market. In the basic specification *Source:* SIAB.